Bond Graphs

A graphical language for the analysis of multiphysical systems

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Slide deck 1: the language

Course outline

- **Bond graph objectives**
- **The bond graph language**
  - Bonds and power variables: the physical analogy
  - Elements
- Practice: reading & creating bond graphs
- Causality and derivation of mathematical models

Bond graph objectives

- a simple unified graphical language for many physical domains
- acausal models
  - to preserve the physical structure of the real physical system
  - which highlight energy exchange
- but with the (optional) superposition of a *computational causality* information

Bond graph model structure is *hybrid*

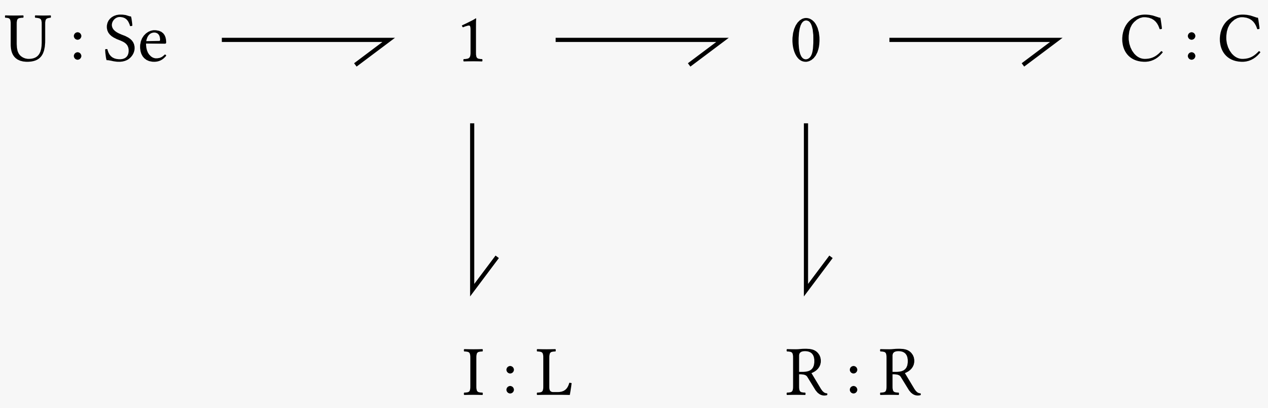
Two extremal structures of model:

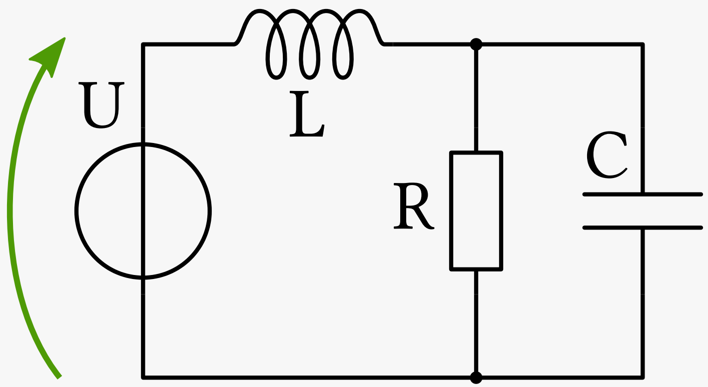
1. Block diagrams, with very clear computational structure
   - but lost physical structure
2. Physical network-type diagrams (electrical, mechanical)
   - but no computational information

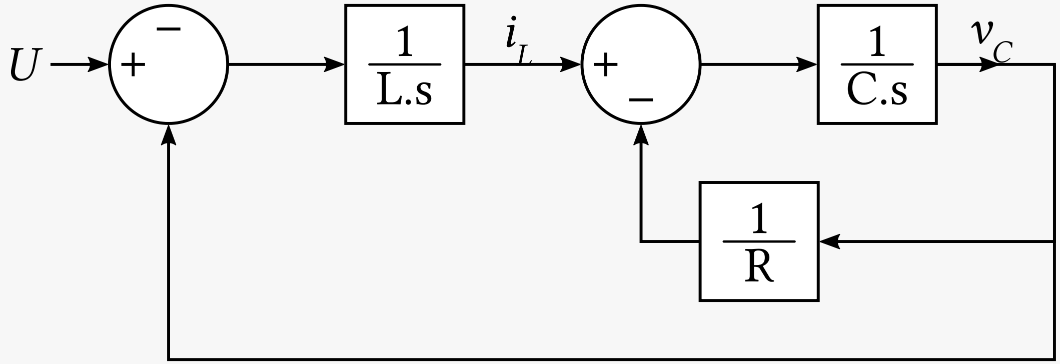
$$
\begin{tikzcd}
\mathrm{U : Se} \arrow[r, harpoon'] & 1 \arrow[r, harpoon'] \arrow[d, harpoon] & 0 \arrow[r, harpoon'] \arrow[d, harpoon] & \mathrm{C : C} \\
 & \mathrm{I : L} & \mathrm{R : R} &
\end{tikzcd}
$$

Model comparisons

BG, Circuit (acausal), Block diagram (causal)





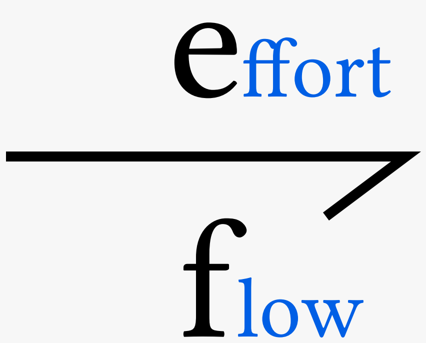


The bond graph language

- Bonds & power variables
- Elements

Bonds

Bonds model the physical interaction of **two** elements which **exchange energy**.



The interaction happens through two generalized physical variables: *effort* & *flow*, collectively named the “power variables”.

Physical analogy in bond graphs

Each physical domain has a specific choice for the generalized *effort* and *flow* variables of each bond:

	Effort e	**Flow f**
Translational mechanics	Force F (N)	Velocity v (m/s)
Rotational mechanics	Torque Γ (N.m)	Angular velocity ω (rad/s)
Electricity	Voltage u (V)	Current i (A)
Thermal transfers	Temperature T (K)	Entropy flow rate $\dot{S}$ (J/K/s)
Hydraulic	Pressure P (N/m²)	Volume flow rate Q_v (m³/s)

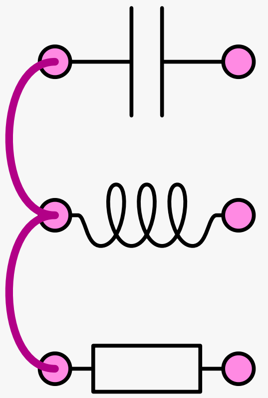
Property: the two variables of each bond are chosen such that:

$$Effort \times Flow = Power \text{ (Watt)}$$

Reminder: Modelica's physical analogy

Modelica's analogy is based on the port's *connection* behavior

	Potential	Flow
Translat. mech.	**Position** s (m)	Force f (N)
Rotational mech.	Angular **position** φ (rad)	Torque Γ (N.m)
Electricity	Voltage u (V)	Current i (A)
Thermal transfers	Temperature T (K)	**Heat** flow Q (J/s=W)
Hydraulic	Pressure P (N/m²)	**Mass** flow rate Q_m (kg/s)



$$v_1=v_2=v_3$$
$$i_1+i_2+i_3=0$$

⚠️ Differences with bond graph:

- Force and Torque are **switched**: BG's Effort ⟶ Modelica's Flow
- Different vocabulary in **bold**: in particular position vs speed

Comparison with Modelica's analogy

BG & Modelica:

- both introduce an analogy between variables across different physical domains
- but using a different **classification** & **vocabulary**, because each is built on different foundations

BG's analogy: group variables as *"effort"* or *"flow"*

- by *preserving common physical sense* (ex.: voltage $\leftrightarrow$ force, current $\leftrightarrow$ speed)
- with constraint $e \times f = Power$

Modelica's analogy: group variables as *"potential"* or *"flow"*

- by *preserving the connection topology* of graphical diagrams (ex.: voltage $\leftrightarrow$ position because both are equal at interconnection of ports)
- (with no constraint on the product $e \times f$)

Relation between BG's and Modelica's analogies

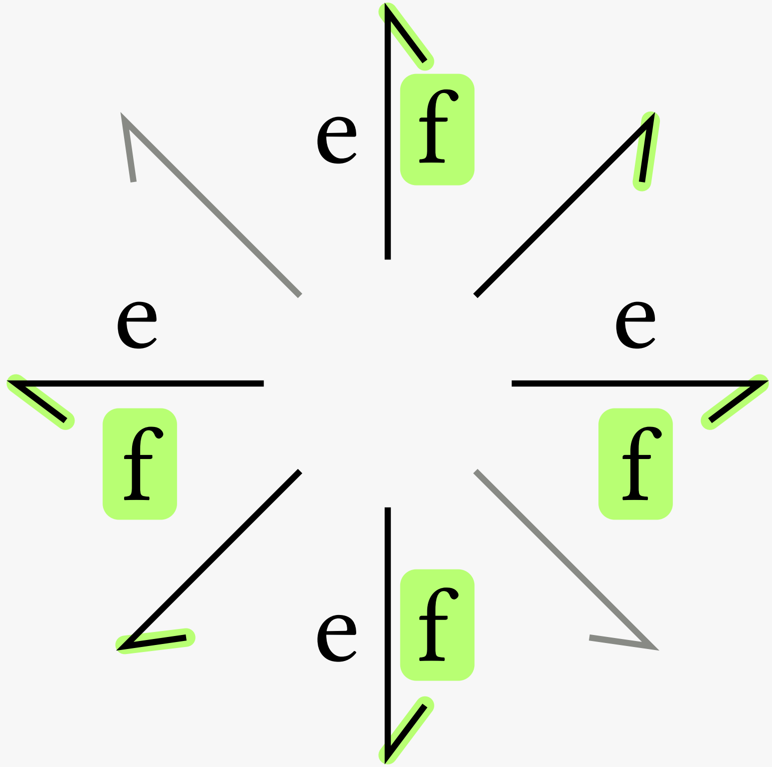
Domain	Mechanics	Others (e.g. electricity)
BG effort	Mod flow	Mod potential
BG flow	Mod der(potential)	Mod flow (*)

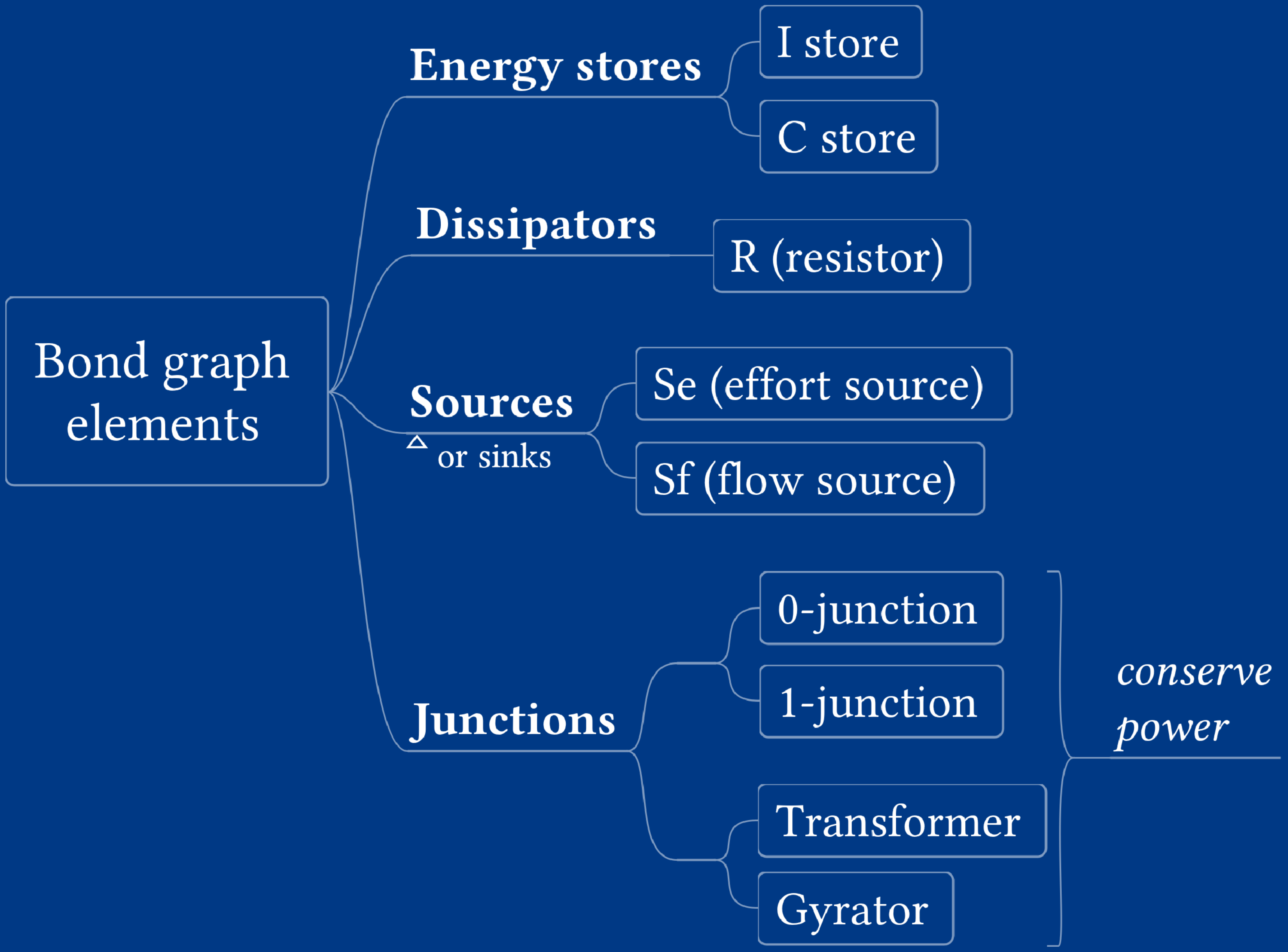
Observations:

- for all domains except mechanics, we have:
  - BG effort = Modelica potential
  - BG flow = Modelica flow (*)
- but for mechanics:
  - it's reversed
  - and we have an extra derivative: speed = der(position)
- (*) thermal domain is an exception : heat flow (J/s) *vs* entropy flow (J/K/s)

Bonds

- Exchange of energy between elements
- Half arrow = direction of positive power flow
- Drawing conventions:
  - e above/left, f below/right
  - half arrow on the side of the flow (i.e. below/right)




Bond graph elements
Energy stores
I store
C store
Dissipators
R (resistor)
Sources
or sinks
Se (effort source)
Sf (flow source)
Junctions
0-junction
1-junction
Transformer
Gyrator
conserve power

Junctions

Unlike in network-type diagrams, the connection of elements in not achieved with the topology of links (e.g. loops of wires),
but using explicitly one of the two junction elements:

- “0” junction: common effort
- “1” junction: common flow

Also in the junction category: tranformers and gyrators

Common property: **instantaneous power is conserved**

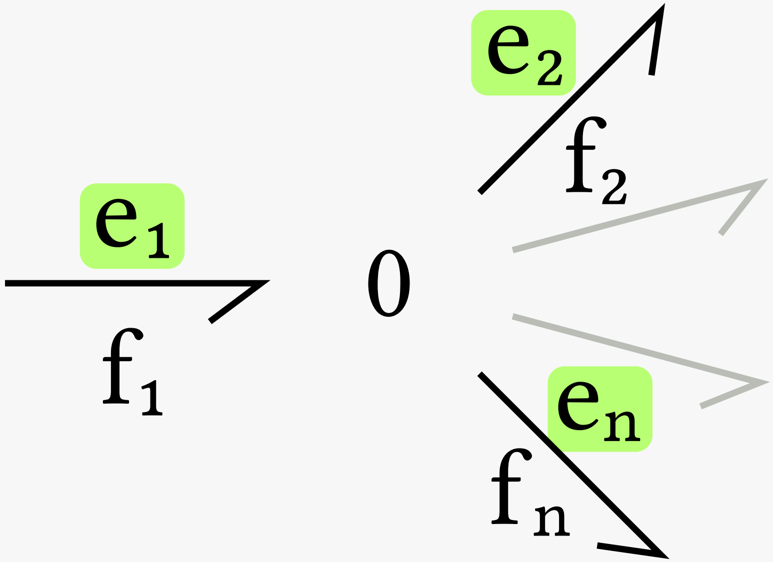
0 junction

“Common effort” junction:

$$e_1 = e_2 = \ldots = e_n$$

Flows are distributed (incoming sum = outgoing sum), according to the *orientation* of the bonds.
On the example:

$$f_1 = f_2 + \ldots + f_n$$



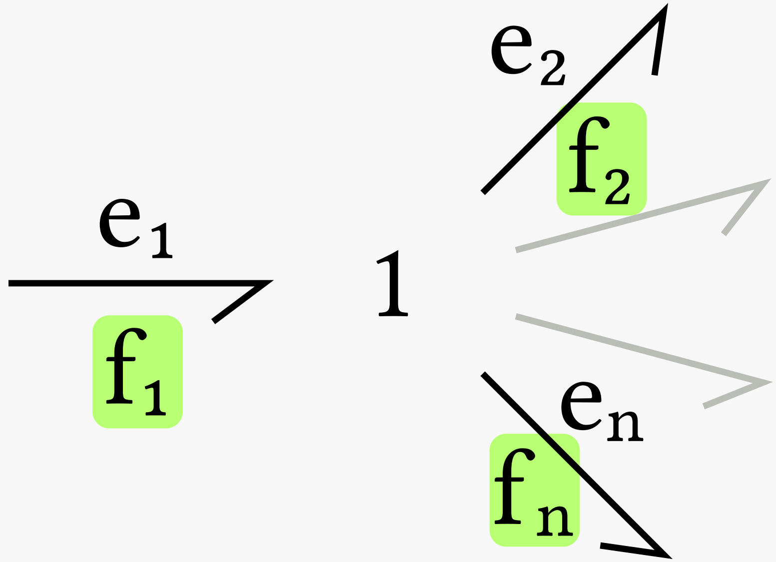
1 junction

"Common flow" junction:

$$f_1 = f_2 = \ldots = f_n$$

Efforts are distributed (incoming sum = outgoing sum), according to the *orientation* of the bonds.
On the example:

$$e_1 = e_2 + \ldots + e_n$$

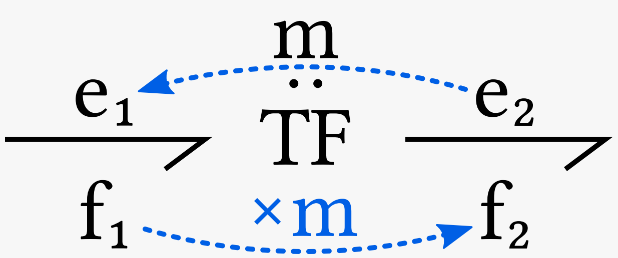


Two-port junctions: transformers & gyrators

Transmit power with a scaling of efforts & flows:

- in the same domain (unitless scaling)
- between two domains (scaling with a physical unit)

Transformer (TF)



$$f_2 = m.f_1$$

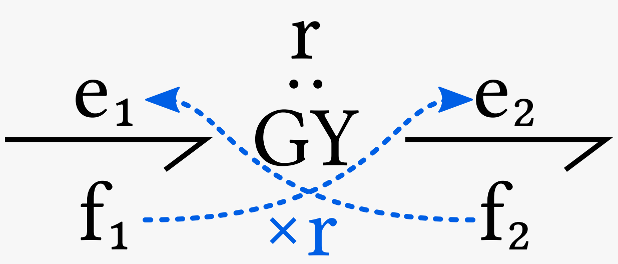
$$m.e_2 = e_1$$

m is the transformer ratio

Examples:

- Mechanical gear pair: $\omega_2 = (r_2/r_1).\omega_1$
- Cable — Pulley: $v = r.\omega$
- Electrical **transformer**: $v_1 = m.v_2$ (⚠️ inverted definition of the transformer ratio)

Gyrator (GY)



$$e_2 = r.f_1$$

$$e_1 = r.f_2$$

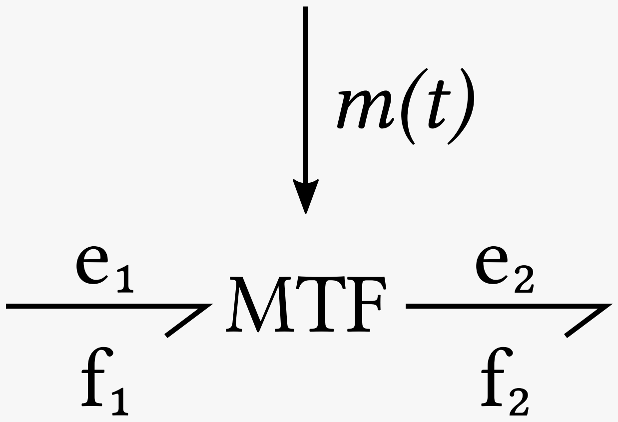
r is the gyrator ratio

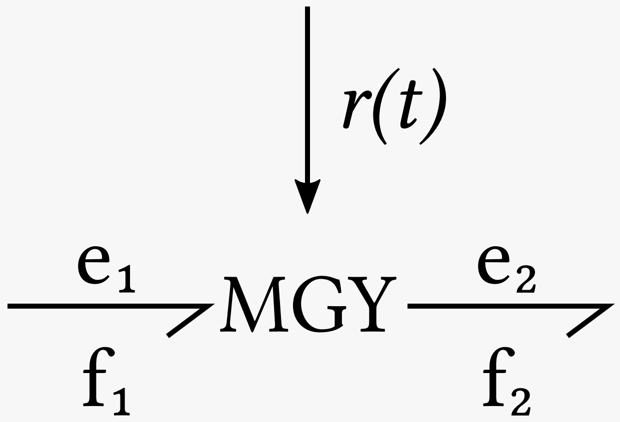
Example:

- EMF of a DC motor: $e = K.\Omega$ and $C = K.i$

Modulation of Transformers (Gyrators)

The transformer (gyrator) can be *modulated by a signal* (using a *signal arrow* $\longrightarrow$):





Examples: crank-slider mechanism, averaged DC-DC converter.

One-port elements

There are 3 basic energy consuming/storing devices:

- Dissipator: **R**esistor
- Energy stores:
  - **C** store, also called **C**ompliance or **C**apacitor
  - **I** store, also called **I**nertia

In addition, there are two sources: **Se** (effort) and **Sf** (flow source)

Remark: in electricity, *one-port* element = device with *two* electrical pins

Resistor (R)

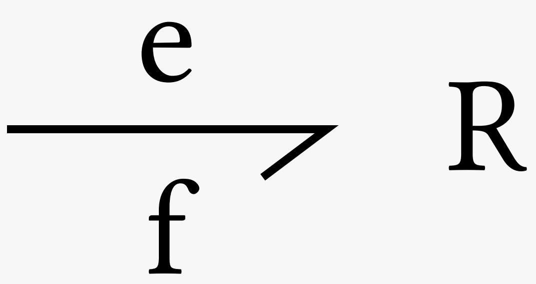
Relation (linear case):

$$e = R.f$$

Property: power is irreversibly dissipated (as heat)

Examples:

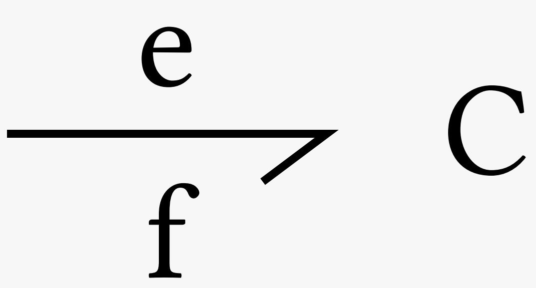
- Electrical resistor: $u = R.i$
- Mechanical damper: $f = d.v$



C energy store (also called Compliance or Capacitor)

Using the “generalized displacement” q (an “energy” variable):

$$q = \int f.dt$$



C store relation:

$$q = \Phi_C(e)$$

Linear C store:

$$q = C.e$$

$\longrightarrow$ consequence: $f = C.de/dt$

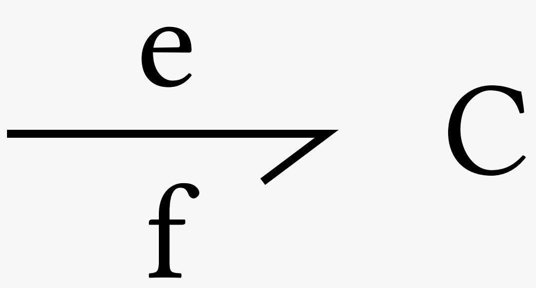
C store examples (linear)

Mechanics:

- displacement = (kinematic) displacement $x = \int v.dt$
- relation: $x = (1/k).f$
- C store = **spring**

Electricity:

- displacement = charge $q = \int i.dt$ (Coulomb)
- relation $q = C.u$
- C store = (electrical) **capacitor**



I energy store (also called Inertia)

Using the "generalized momentum" p (an "energy" variable):

$$p = \int e.dt$$

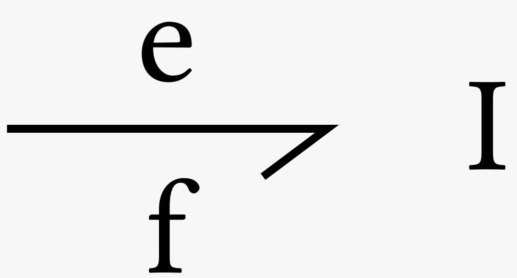
I store relation:

$$p = \Phi_I(f)$$

Linear I store:

$$p = I.f$$

$\rightarrow$ consequence: $e = I.df/dt$



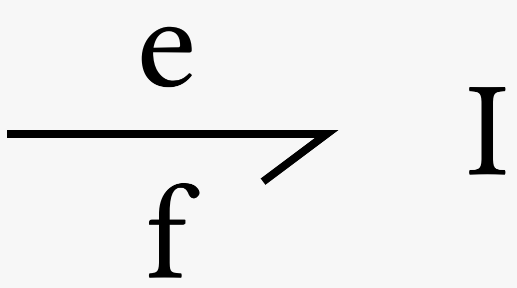
I store examples (linear)

Mechanics:

- momentum = mechanical momentum $p = \int f.dt$
- relation: $p = m.v$, that is $f = m.dv/dt$ (inertial force)
- I store = mechanical **inertia**

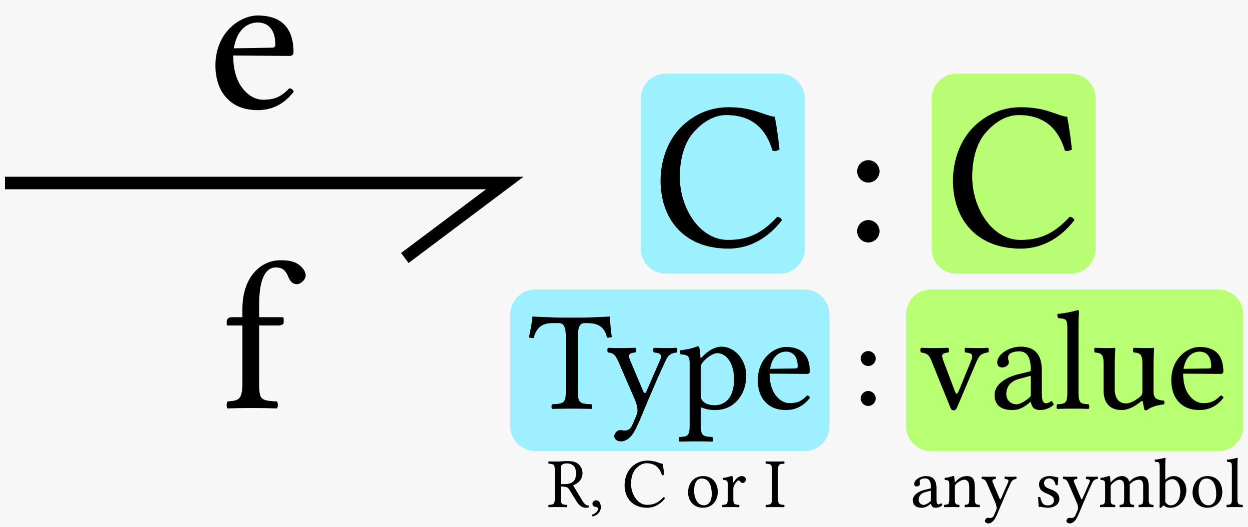
Electrity:

- momentum = magnetic flux linkage $\lambda = \int u.dt$ (V.s = Wb)
- relation: $\lambda = L.i$, that is $v = L.di/dt$
- I store = **inductor**



Parametrization of R, C, I elements

The value of a linear R/C/I element is appended with the notation “: x”.



Same notation is used for the value of a source (next slide).

Sources

Sources either impose the effort (**Se**) or the flow (**Sf**).

Se examples:

- Electricity: voltage source
- Mechanics: imposed torque or force

Sf examples:

- Electricity: current source
- Mechanics: imposed speed