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Optical Phase Conjugation

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Download this document: http://moodle.univ-metz.fr/
Useful reading... [YY84, Yar97, San99]


...and many others.
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A peculiar phenomenon
Discovered in the early 70\textsuperscript{ies}

Use Non Linear Material
- Third order: non zero $\chi_3$
  - Photorefractivity
  - Stimulated Brillouin Scattering
  - Stimulated Raman Scattering
- Four wave mixing

Wavefront correction
- Distorted incident beam
- Reflected back \textit{as is}
- Distortion corrected

\textbf{Figure:} Phase conjugation principle.
\textit{Source:} Wikipedia
Principle and application of phase conjugation

Generation of phase conjugate waves

Self Pumping and Holography

Experiment

Beams are reflected back *as if time was reversed*

Images source:
http://sharp.bu.edu/~slehar/PhaseConjugate/PhaseConjugate.html

In the experiment, Incident wavefronts are reflected back exactly. Back and forth wavefronts are identical.
Attractive applications
All based on wavefront distortion correction

Phase conjugation applications

- All optical image transmission through fibers
- Distortion correction in high power lasers
- Dynamic wave front correction for optical sensors
- Dynamic Holography
- ...
**A phase conjugate wave travels time the wrong way**

Phase conjugation is also known as *time reversal*

---

**Take some input monochromatic wave**

\[ E_1 = \Re\left[ \psi (r) \exp (i (\omega t - kz)) \right] = \Re\left[ \psi (r) \right] \cos (\omega t - kz) \]

**Take the phase conjugate over space only**

- \[ E_2 = \Re\left[ \overline{\psi (r)} \exp (i (-kz)) e^{i\omega t} \right] \]
- \[ E_2 = \Re\left[ \overline{\psi (r)} \exp (i (\omega t + kz)) \right] \]
- \[ E_2 = \Re\left[ \overline{\psi (r)} \right] \cos (\omega t + kz) \]
- \[ E_2 = \Re\left[ \psi (r) \right] \cos (-\omega t - kz) \]
Principle and application of phase conjugation

Generation of phase conjugate waves

Self Pumping and Holography

Distortion correction theorem

The distortion correction theorem

If a backward traveling wave is phase conjugate somewhere then it is everywhere

Take a paraxial forward propagating wave

- Expressed as: \( E_1 (r, t) = \psi_1 (r) e^{i(\omega t - kz)} \)
- Obeys the wave equation: \( \Delta E_1 + \omega^2 \mu_0 \varepsilon (r) E_1 = 0 \)
- In the paraxial limit: \( \Delta \psi_1 + \left[ \omega^2 \mu_0 \varepsilon (r) - k^2 \right] \psi_1 - 2i k \frac{\partial \psi_1}{\partial z} = 0 \)
- Conjugate equation: \( \Delta \bar{\psi}_1 + \left[ \omega^2 \mu_0 \varepsilon (r) - k^2 \right] \bar{\psi}_1 + 2i k \frac{\partial \bar{\psi}_1}{\partial z} = 0 \)

Had we taken a backward propagating wave

- Expressed as: \( E_2 (r, t) = \psi_2 (r) e^{i(\omega t + kz)} \)
- Paraxial equation \((z \rightarrow -z)\):
  \( \Delta \psi_2 + \left[ \omega^2 \mu_0 \varepsilon (r) - k^2 \right] \psi_2 + 2i k \frac{\partial \psi_2}{\partial z} = 0 \)

Same second order linear differential equations for loss-less media

\( \varepsilon (r) \in \mathbb{R} \Rightarrow \left[ \psi_2 (z = 0) = a.\bar{\psi}_1 (z = 0) \Leftrightarrow \forall z, \psi_2 (z) = a.\bar{\psi}_1 (z) \right] \)
Four wave mixing
Third Order Non Linear Optics

Flashback: Second Order

- Relies on $\chi_2 : P_{NL} \propto E^2$
- Two waves mix to generate a third one
  - $\omega_1 \pm \omega_2 \rightarrow \omega_3$
- Second Harmonic Generation, Optical Parametric Amplification, Optical Parametric Oscillation...

Third order

- Relies on $\chi_3 : P_{NL} \propto E^3$
- Three waves mix to generate a fourth one
  - $\omega_1 \pm \omega_2 \pm \omega_3 \rightarrow \omega_4$
- Phase conjugation for $\omega_4 = \omega_1 + \omega_2 - \omega_3$? Let's see...
Non Linear Polarization Development

Non Linear Polarization $P_{NL}$

### General polarization development

$[P]_i = \varepsilon_0 [\chi]_{ij} [E]_j + 2 [d]_{ijk} [E]_j [E]_k + 4 [\chi]_{ijkl} [E]_j [E]_k [E]_l$

### Third order non linear development\(^1\) around $\omega_4 = \omega_1 + \omega_2 - \omega_3$

$[P_{NL}]_i (\omega_4) = 6 [\chi]_{ijkl} [E]_j (\omega_1) [E]_k (\omega_2) [E]_l (\omega_3) e^{i(\omega_4 t + k_4 r)}$

### Degenerate Four wave mixing : $\omega = \omega + \omega - \omega$

$[P_{NL}]_i (\omega) = 6 [\chi]_{ijkl} [E]_j (\omega) [E]_k (\omega) [E]_l (\omega) e^{i(\omega t + kr)}$

---

\(^1\)As an exercise, you can multiply, sum-up and keep only $\omega_4$ related terms... and find $k_4$
Degenerate Four Wave mixing configuration

- $A_1 = \overline{A_2}$ intense plane pumps
- $A_3$ is the signal
- We seek $A_4$
Let us start with the standard non linear wave equation

\[ \Delta E - \mu_0 \varepsilon \frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial^2 P_{NL}}{\partial t^2} \]

**Signal \( A_3 \) propagation**

- Each wave has its own direction and polarization
- They can be treated separately

With \( \Delta E_3 = \mathcal{R}e \left[ \left( -k^2 A_3 - 2ik \frac{\partial A_3}{\partial z} + \frac{\partial^2 A_3}{\partial z^2} \right) e^{i(\omega t - kz)} \right] \)

\[ \mathcal{R}e \left[ \left( -2ik \frac{\partial A_3}{\partial z} + \frac{\partial^2 A_3}{\partial z^2} \right) e^{i(\omega t - kz)} \right] = \mu_0 \frac{\partial^2 P_{NL}}{\partial t^2} \]

**Evaluation of the non linear polarization \( P_{NL} \)**

Let us take a look at the terms which involve \( e^{i(\omega t \pm kz)} \)
Stripping the non linear polarization to useful terms
Keeping only the relevant terms which contain $e^{i(\omega t \pm kz)}$

**Expansion of third order non linear polarization**

$$[P_{NL}]_i = \Re e \left[ 6 \left( \begin{array}{c} [\chi]_{ijkl} [A_1]_j [A_2]_k [A_4]_l \\ + [\chi]_{ijji} [A_1]_j \frac{[A_1]}{[A_2]} [A_3]_i \\ + [\chi]_{ikki} [A_2]_k \frac{[A_2]}{[A_3]} [A_3]_i \end{array} \right) e^{i(\omega t - kz)} \right]$$

**Simplifying assumptions**

- Intense pump beam terms are dominant
- Polarizations are
  - either all the same, only $[\chi]_{iii}$ involved
  - or $(A_1/A_2) \perp (A_3/A_4)$, only $[\chi]_{ijji}$, $i \neq j$ involved
- $[P_{NL}]_i = \chi^{(3)} \left[ (||A1||^2 + ||A2||^2) A_3 + A_1 A_2 \overline{A_4} \right] e^{i(\omega t - kz)}$
  - $\chi^{(3)} = 6[\chi]_{iii}$ or $\chi^{(3)} = 6[\chi]_{ijji}$
Principle and application of phase conjugation

Generation of phase conjugate waves

Self Pumping and Holography

Four wave mixing coupled mode formulation

Resulting coupled wave propagation equation

Simplified coupling equations

\[ \frac{\partial A_4}{\partial z} = i \kappa A_4 \]

obtained through the same kind of derivation

Further simplification

Homogeneous refraction index modulation : Kerr effect

Set \( \kappa = \frac{\omega}{2} \sqrt{\varepsilon \chi (3) A_1 A_2} \)

Simple phase factor change

Remove it from equation \( A'_i = A_i e^{-i \omega z} \sqrt{\varepsilon \chi (3)} \)

\[ \frac{\partial A_3}{\partial z} = -i \omega \sqrt{\frac{\mu_0}{\varepsilon \chi (3)}} (|A_1|^2 + |A_2|^2) A_3 + A_1 A_2 A_4 \]

Simplified coupling equations
Conjugate wave amplitude

**General solution**
Boundary conditions at $z = 0$ and $z = L$

$\mathbf{A}_3$ is forward propagating

\[
A_3'(z) = -i \frac{|\kappa| \sin(|k|z)}{\kappa \cos(|k|L)} A_4'(L) + \frac{\cos(|k|(z - L))}{\cos(|k|L)} A_3'(0)
\]

$A_4'(z) = \frac{\cos(|k|z)}{\cos(|k|L)} A_4'(L) + i \frac{|\kappa| \sin(|k|(z - L))}{|k| \cos(|k|L)} A_3'(0)$

$\mathbf{A}_4$ is backward propagating

**One beam experiment**

$A_4'(L) = 0$

\[
A_3'(L) = \frac{A_3'(0)}{\cos(|k|L)}
\]

Coherent amplifier

\[
A_4'(0) = -i \frac{|\kappa|}{|\kappa|} \tan(|\kappa|L) \overline{A_3'(0)}
\]

Reflectivity can exceed 1
One Beam experiment and phase factor

One beam experiment

- \( A_3'(L) = \frac{A_3'(0)}{\cos(|k|L)} \)
- \( A_4'(0) = -i \frac{\kappa}{|\kappa|} \tan(|\kappa|L) A_3'(0) \)

Coherent amplifier

Reflectivity can exceed 1

What if \( \cos(|k|L) = 0 \)?

- Infinite gain
- \( A_3 \) and \( A_4 \) start from noise
- Spontaneous oscillations
Four Wave Mixing is Real Time Holography

Write Hologram

\[ T \propto \|A_1 + A_3\|^2 = \|A_1\|^2 + \|A_3\|^2 + A_1A_3 + A_3A_1 \]

Read Hologram with \( A_2 = \overline{A_1} \)

\[ A_4 \propto TA_2 = (\|A_1\|^2 + \|A_3\|^2)A_2 + A_2A_1\overline{A_3} + A_2A_3\overline{A_1} \]
Total internal reflection

Figure: Beam fanning in photorefractive Baryum Titanate