UE SPM-PHY-S07-101
Polarization Optics

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Further reading

[Hua94, K8̈5]

- S. Huard.
  Polarisation de la lumière.
  Masson, 1994.
- G. P. Können.
  Polarized light in Nature.
  Cambridge University Press, 1985.

Course Outline

The vector nature of light

Optical wave can be polarized, sound waves cannot

The scalar monochromatic plane wave

The electric field reads: $A\cos(\omega t - kz - \varphi)$

A vector monochromatic plane wave

- Electric field is orthogonal to wave and Poynting vectors
- Lies in the wave vector normal plane
- Needs 2 components
  - $E_x = A_x \cos(\omega t - kz - \varphi_x)$
  - $E_y = A_y \cos(\omega t - kz - \varphi_y)$

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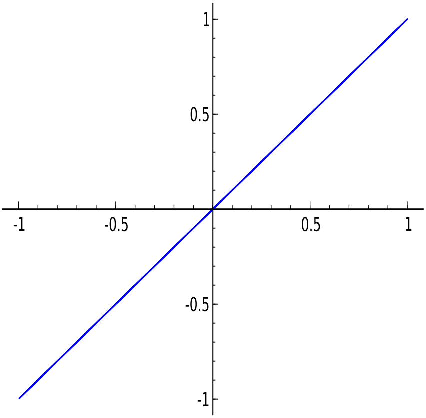
Linear and circular polarization states

In phase components $\varphi_y = \varphi_x$



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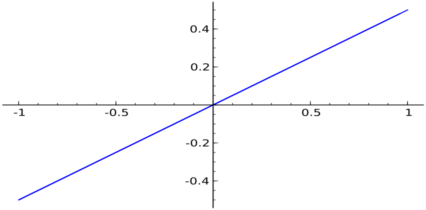

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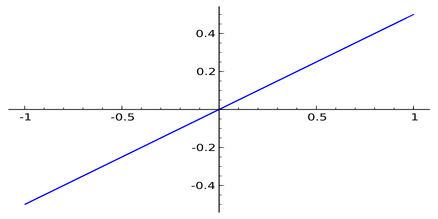
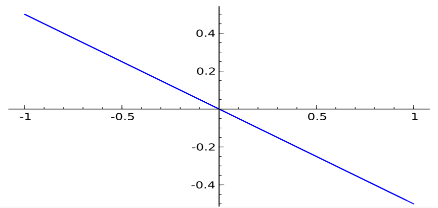


π shift $\varphi_y = \varphi_x + \pi$



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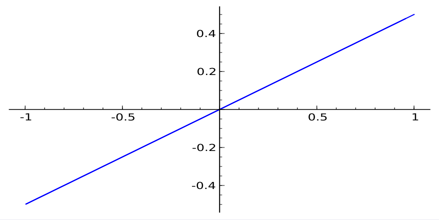

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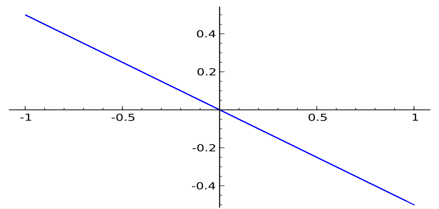
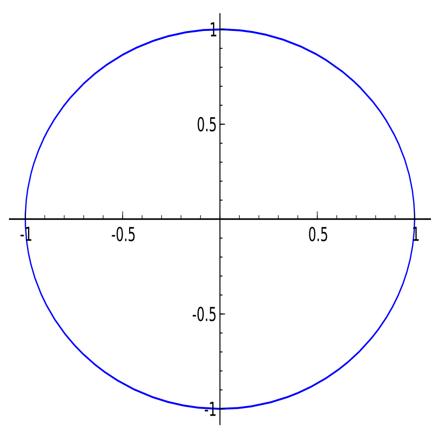


$\pi/2$ shift $\varphi_y = \varphi_x \pm \pi/2$



Left or Right

Linear and circular polarization states

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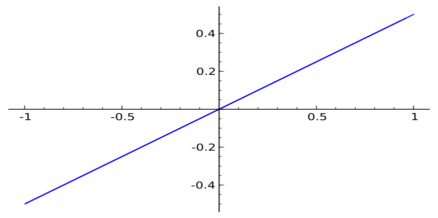

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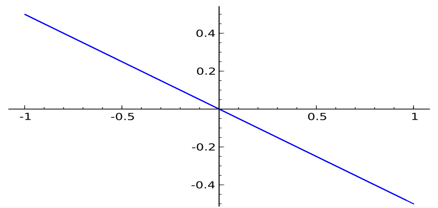
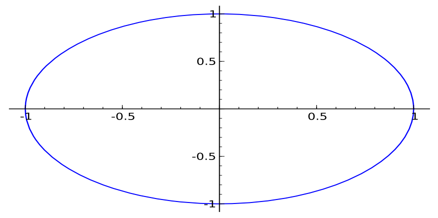


$\pi/2$ shift $\varphi_y = \varphi_x \pm \pi/2$



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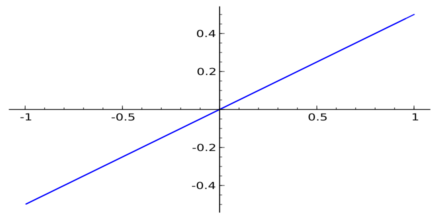

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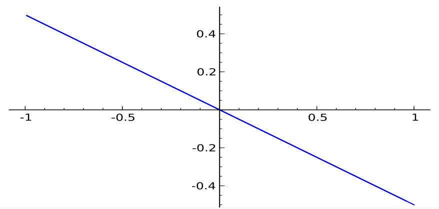

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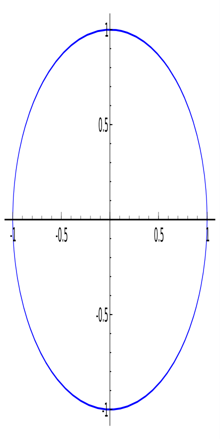
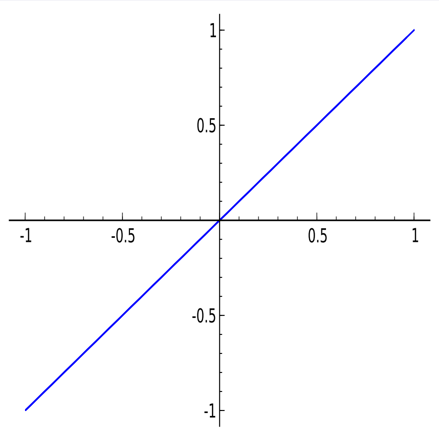


Left or Right

The elliptic polarization state

The polarization state of ANY monochromatic wave

$\varphi_y - \varphi_x = \pm 0$



Electric field

- $E_x = A_x \cos\left(\omega t - kz - \varphi_x\right)$
- $E_y = A_y \cos\left(\omega t - kz - \varphi_y\right)$

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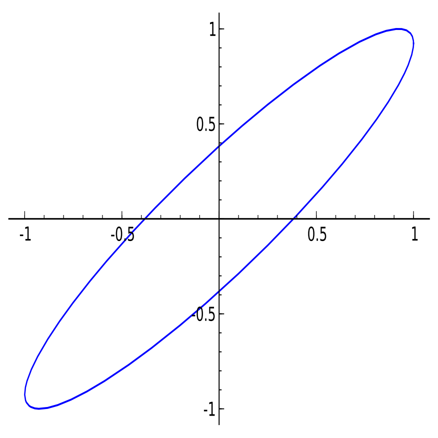
$\varphi_y - \varphi_x = \pm\pi/8$



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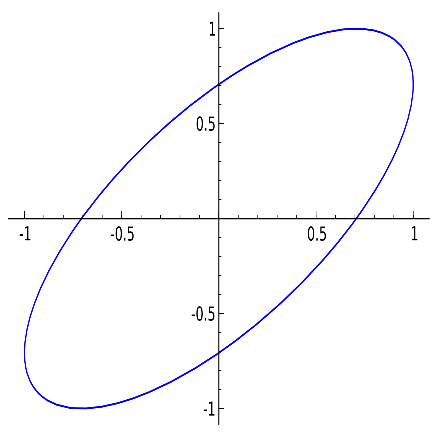
$\varphi_y - \varphi_x = \pm\pi/4$



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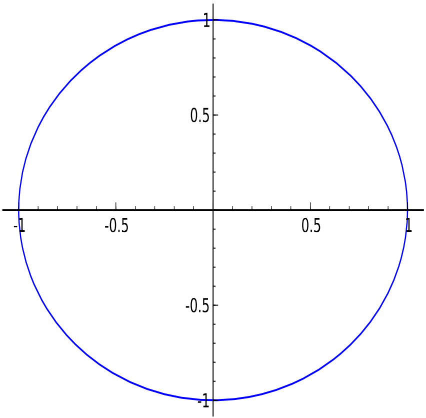

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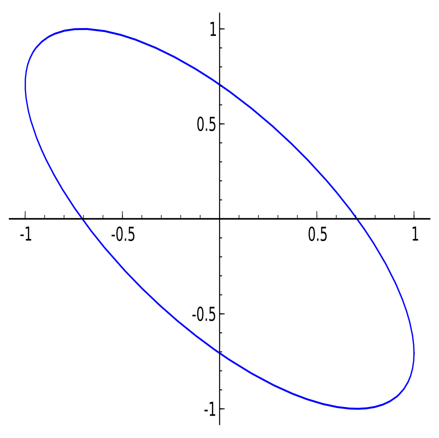
$\varphi_y - \varphi_x = \pm 3\pi/4$



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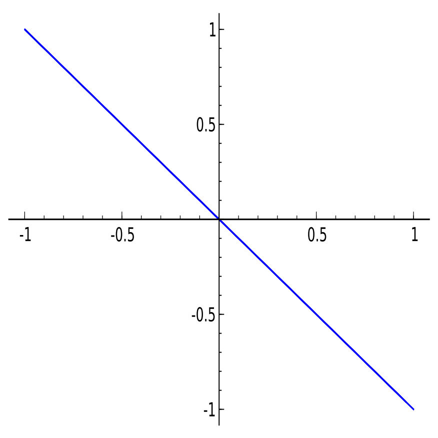

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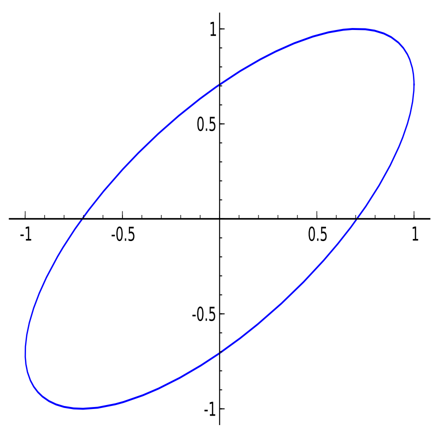

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4 real numbers

- A_x,φ_x
- A_y,φ_y

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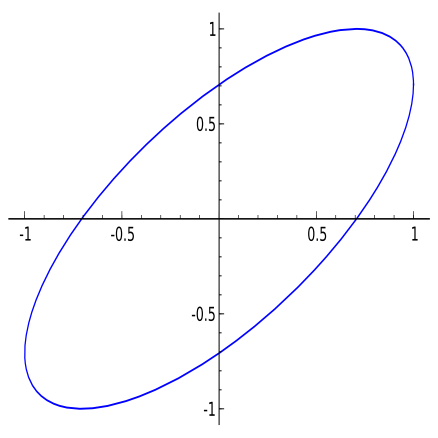

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4 real numbers

- A_x, φ_x
- A_y, φ_y

2 complex numbers

- $A_x \exp(i\varphi_x)$
- $A_y \exp(i\varphi_y)$

Polarization states are vectors

Monochromatic polarizations belong to a 2D vector space based on the Complex Ring

ANY elliptic polarization state $\Longleftrightarrow$ Two complex numbers

A set of two ordered complex numbers is one 2D complex vector

Canonical Basis

$$\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$$

Link with optics ?

- These two vectors represent two polarization states
- We must decide which ones !

Polarization Basis

Two independent polarizations :

- Crossed Linear
- Reversed circular
- ...
- YOUR choice

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Examples : Linear Polarizations

Canonical Basis Choice

- $\begin{bmatrix}1\\0\end{bmatrix}$: horizontal linear polarization
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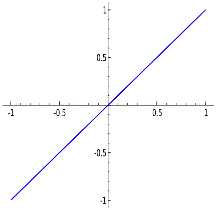
Tilt $\theta = \pi/4$

$$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$



Examples : Linear Polarizations

Canonical Basis Choice

- $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$: horizontal linear polarization
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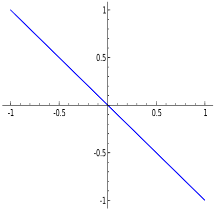
Tilt $\theta = 3\pi/4$

$$\frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$



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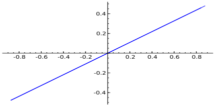

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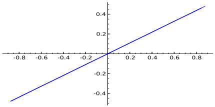


Linear polarization Jones vector

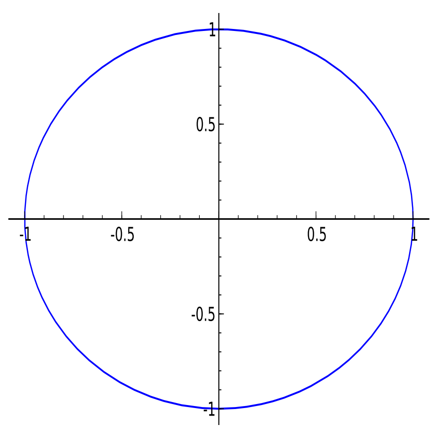
- Linear Polarization : two in phase components
- Two real numbers

In a linear polarization basis

Examples : Circular Polarizations

In the same canonical basis choice : linear polarizations

$\varphi_y - \varphi_x = \pm\pi/2$



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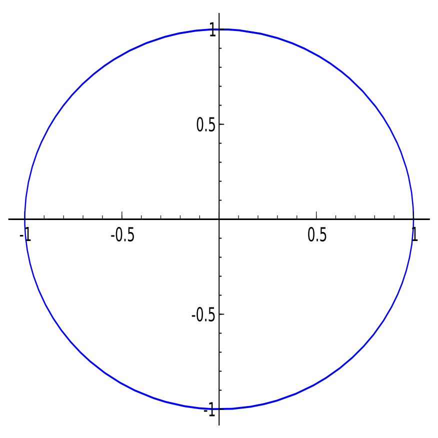

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About changing basis

A polarization state Jones vector is basis dependent

Some elementary algebra

- The polarization vector space dimension is 2
- Therefore : two non colinear vectors form a basis
- Any polarization state can be expressed as the sum of two non colinear other states
- Remark : two colinear polarization states are identical

Homework

Find the transformation matrix between between the two following bases :

- Horizontal and Vertical Linear Polarizations
- Right and Left Circular Polarizations

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Relationship between Jones and Poynting vectors

Jones vectors also provide information about intensity

Choose an orthonormal basis (J_1, J_2)

- Hermitian product is null : $\overline{J_1} \cdot J_2 = 0$
- Each vector norm is unity : $\overline{J_1} \cdot J_1 = \overline{J_2} \cdot J_2 = 1$

Hermitian Norm is Intensity

Simple calculations show that :

- If each Jones component is one complex electric field component
- The Hermitian norm is proportional to beam intensity

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Polarization as a unique complex number

If the intensity information disappears, polarization is summed up in one complex number

Rule out the intensity

Norm the Jones vector to unity

Put 1 as first component

- Multiplying Jones vector by a complex number does not change the polarization state
- Norm the first component to 1 : $\begin{bmatrix} 1 \\ \xi \end{bmatrix}$
- The sole ξ describes the polarization state

Choose between the two

Either you norm the vector, or its first component. Not both !

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The Stokes parameters

A set of 4 dependent real parameters that can be measured

Sample Jones Vector

$$\begin{bmatrix} A_x \exp\left(+i\varphi/2\right) \\ A_y \exp\left(-i\varphi/2\right) \end{bmatrix}$$

P_0 Overall Intensity

$$P_0 = A_x^2 + A_y^2 = I$$

P_1 Intensity Différence

$$P_1 = A_x^2 - A_y^2 = I_x - I_y$$

P_2 $\pi/4$ Tilted Basis

$$J_{\pi/4} = \frac{1}{\sqrt{2}} \begin{bmatrix} A_x e^{-i\varphi/2} + A_y e^{+i\varphi/2} \\ A_x e^{-i\varphi/2} - A_y e^{+i\varphi/2} \end{bmatrix}$$

$$P_2 = I_{\pi/4} - I_{-\pi/4} = 2A_x A_y \cos\left(\varphi\right)$$

P_3 Circular Basis

$$J_{\mathrm{cir}} = \frac{1}{\sqrt{2}} \begin{bmatrix} A_x e^{-i\varphi/2} - iA_y e^{+i\varphi/2} \\ A_x e^{-i\varphi/2} + iA_y e^{+i\varphi/2} \end{bmatrix}$$

$$P_3 = I_{\mathrm{L}} - I_{\mathrm{R}} = 2A_x A_y \sin\left(\varphi\right)$$

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P_3 Circular Basis

$$J_{\mathrm{cir}} = \frac{1}{\sqrt{2}} \begin{bmatrix} A_x e^{-i\varphi/2} - iA_y e^{+i\varphi/2} \\ A_x e^{-i\varphi/2} + iA_y e^{+i\varphi/2} \end{bmatrix}$$

$$P_3 = I_{\mathrm{L}} - I_{\mathrm{R}} = 2A_x A_y \sin\left(\varphi\right)$$

The Stokes parameters

A set of 4 dependent real parameters that can be measured

Sample Jones Vector

$$\begin{bmatrix} A_x \exp\left(+i\varphi/2\right) \\ A_y \exp\left(-i\varphi/2\right) \end{bmatrix}$$

P_0 — Overall Intensity

$$P_0 = A_x^2 + A_y^2 = I$$

P_1 — Intensity Différence

$$P_1 = A_x^2 - A_y^2 = I_x - I_y$$

P_2 — $\pi/4$ Tilted Basis

$$J_{\pi/4} = \frac{1}{\sqrt{2}} \begin{bmatrix} A_x e^{-i\varphi/2} + A_y e^{+i\varphi/2} \\ A_x e^{-i\varphi/2} - A_y e^{+i\varphi/2} \end{bmatrix}$$

$$P_2 = I_{\pi/4} - I_{-\pi/4} = 2A_xA_y\cos\left(\varphi\right)$$

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4 dependent parameters

$$P_0^2 = P_1^2 + P_2^2 + P_3^2$$

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Homework

Find the reverse relationship : φ,A_x, A_y from the Stokes parameters

The Poincare Sphere

Polarization states can be described geometrically on a sphere

Recall the Stokes parameters

$$P_0^2 = P_1^2 + P_2^2 + P_3^2$$

Normalized Stokes parameters

$$S_i = P_i / P_0$$

Unit Radius Sphere

$$\sum_{i=1}^{3} S_i = 1$$

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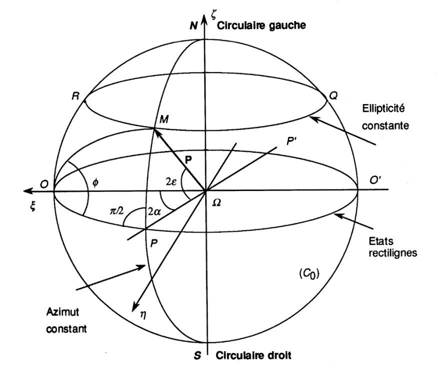
(S_1, S_2, S_3) on a unit radius sphere



Figures from [Hua94]

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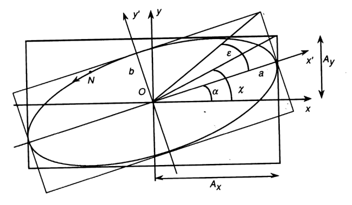
General Polarisation



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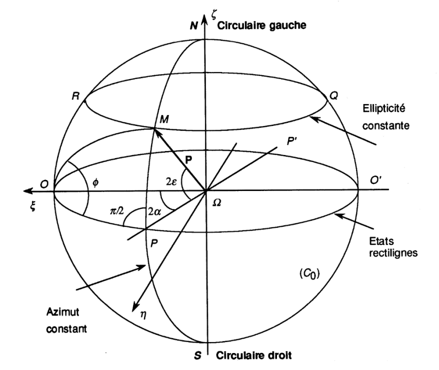

Figures from [Hua94]

Eigen Polarization states

Polarization states that do not change after propagation in an anisotropic medium

Eigen Polarization states

- Do not change
- Except for Intensity

Linear Anisotropy Eigen Polarizations

- Quarter and half wave plates and Birefringent materials
- Eigen Polarizations are linear along the eigen axes

Circular Anisotropy

- Also called optical activity
- e.g. in Faraday rotators and in gyratory non linear crystals
- Linear polarization is rotated by an angle proportional to propagation distance
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Hermitian operator

2 eigen polarization states are orthonormal

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Jones Matrices

2D Linear Algebra to compute polarization propagation through devices

Jones matrices in the eigen basis

- Let λ_1 and λ_2 be the two eigenvalues of a given device
- e.g. for linear anisotropy : $\lambda_i = e^{n_i k_0 \Delta z}$
- Jones Matrix is $\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$

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Homework

Find half and quarter wave plates Jones Matrices

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In another basis

- Let $\overrightarrow{J_1} = \begin{bmatrix} u \\ v \end{bmatrix}$ and $\overrightarrow{J_2} = \begin{bmatrix} -\overline{v} \\ \overline{u} \end{bmatrix}$ be the orthonormal eigen vectors
- $\begin{cases} \mathbf{M}\overrightarrow{J_1} = \lambda_1 \overrightarrow{J_1} \\ \mathbf{M}\overrightarrow{J_2} = \lambda_2 \overrightarrow{J_2} \end{cases} \Rightarrow \mathbf{M} = \begin{bmatrix} \lambda_1 u\overline{u} + \lambda_2 v\overline{v} & (\lambda_1 - \lambda_2)\, u\overline{v} \\ (\lambda_1 - \lambda_2)\, v\overline{u} & \lambda_2 v\overline{u} + \lambda_1 v\overline{v} \end{bmatrix}$

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In another basis use Transformation Matrix

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The particular case of non absorbing devices

Jones matrix is a unitary operator when $\lambda_1 = \lambda_2 = 1$

Nor absorbing neither amplifying devices

- $|\lambda_1| = |\lambda_2| = 1$
- $\mathbf{M} \cdot \overline{\mathbf{M}^t} = \overline{\mathbf{M}^t} \cdot \mathbf{M} = I$
- $\mathbf{M}$ is a unitary operator

Unitary operator properties

- Norm is conserved : Intensity is unchanged after propagation
- Orthogonality is conserved : two initially orthogonal states will remain so after propagation

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Jones Matrix of a polarizer

In its eigen basis

- A polarized is designed for :
  - Full transmission of one linear polarization
  - Zero transmission of its orthogonal counterpart
- Eigen basis Jones matrix : $\mathbf{P_x} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ or $\mathbf{P_y} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

When transmitted polarization is θ tilted

Change base through θ rotation Transformation Matrix

$$\mathbf{R}(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

$$\mathbf{P}(\theta) = \mathbf{R}(\theta) \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \mathbf{R}(-\theta)$$

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Homework

Find again the θ tilted polarizer Jones Matrix by using physics arguments only

Intensity transmitted through a polarizer

From natural or non polarized light

Half the intensity is transmitted

From linearly polarized light

- Transmitted Jones vector in polarizer eigen basis:

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix} = \begin{bmatrix} \cos(\theta) \\ 0 \end{bmatrix}$$

- Transmitted Intensity : $\cos^2(\theta)$ MALUS law

From circularly polarized light

Show that whatever the polarizer orientation, the transmitted intensity is half the incident intensity.

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Linear anisotropy eigen polarization vectors

- Two orthogonal polarization directions
- Two different refraction indexes n_1 and n_2
- Two linear eigen modes along the eigen directions

Jones Matrix in the eigen basis — Express phase delay only

$$\begin{bmatrix} e^{in_1 k\Delta z} & 0 \\ 0 & e^{in_2 k\Delta z} \end{bmatrix} = e^{i\psi} \begin{bmatrix} e^{i\phi/2} & 0 \\ 0 & e^{-i\phi/2} \end{bmatrix} \approx \begin{bmatrix} e^{i\phi/2} & 0 \\ 0 & e^{-i\phi/2} \end{bmatrix}$$

Quarter and Half wave plates — Homework

- Find the Jones Matrices of Quarter and Half wave plates
- Find their action on tilted linear polarization (special case for $\pi/4$ tilt)
- Find their action on circular polarization

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What is circular anisotropy ?

- Two orthogonal circular eigen polarization states
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Jones Matrix in a linear polarization basis — Transformation matrix

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- $\mathbf{P}_{\mathrm{Cir}\to\mathrm{Lin}} M (\mathbf{P}_{\mathrm{Cir}\to\mathrm{Lin}})^{-1} = e^{i\Psi} \begin{bmatrix} \cos(\phi/2) & \sin(\phi/2) \\ -\sin(\phi/2) & \cos(\phi/2) \end{bmatrix}$

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- use $\mathbf{P}_{\mathrm{Cir}\rightarrow\mathrm{Lin}} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ i & -i \end{bmatrix}$ transformation matrix
- $\mathbf{P}_{\mathrm{Cir}\rightarrow\mathrm{Lin}} M (\mathbf{P}_{\mathrm{Cir}\rightarrow\mathrm{Lin}})^{-1} = e^{i\Psi} \begin{bmatrix} \cos(\phi/2) & \sin(\phi/2) \\ -\sin(\phi/2) & \cos(\phi/2) \end{bmatrix}$

Jones Matrix in the circular eigen basis — Express phase delay only

$$\begin{bmatrix} e^{in_L k\Delta z} & 0 \\ 0 & e^{in_R k\Delta z} \end{bmatrix} = e^{i\psi} \begin{bmatrix} e^{i\phi/2} & 0 \\ 0 & e^{-i\phi/2} \end{bmatrix} \approx \begin{bmatrix} e^{i\phi/2} & 0 \\ 0 & e^{-i\phi/2} \end{bmatrix}$$

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Homework

Show that an incoming linear polarisation is simply rotated by an angle proportional to the propagation distance

Jones Matrices Composition

The Jones matrices of cascaded optical elements can be composed through Matrix multiplication

Matrix composition

- If a $\overrightarrow{J_0}$ incident light passes through $\mathbf{M_1}$ and $\mathbf{M_2}$ in that order
- First transmission: $\mathbf{M_1}\overrightarrow{J_0}$
- Second transmission: $\mathbf{M_2}\mathbf{M_1}\overrightarrow{J_0}$
- Composed Jones Matrix : $\mathbf{M_2}\mathbf{M_1}$ Reversed order

Beware of non commutativity

- Matrix product does not commute in general
- Think of the case of a linear anisotropy followed by optical activity
  - in that order
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Stokes parameters for partially polarized light

Generalize the coherent definition using the statistical average intensity

Stokes Vector

$$\overrightarrow{S} = \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} \langle I_x + I_y \rangle \\ \langle I_x - I_y \rangle \\ \langle I_{\pi/4} - I_{-\pi/4} \rangle \\ \langle I_L - I_R \rangle \end{bmatrix}$$

Polarization degree $0 \leq p \leq 1$

$$p = \frac{\sqrt{P_1^2 + P_2^2 + P_3^2}}{P_0}$$

Stokes decomposition — Polarized and depolarized sum

$$\overrightarrow{S} = \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} pP_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix} + \begin{bmatrix} (1-p)\,P_0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \overrightarrow{S_P} + \overrightarrow{S_{NP}}$$

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The Jones Coherence Matrix

Jones Vectors are out

- They describe phase differences
- Meaningless when not monochromatic

Jones Coherence Matrix

- If $\overrightarrow{J} = \begin{bmatrix} A_x(t)\, e^{i\varphi_x(t)} \\ A_y(t)\, e^{i\varphi_y(t)} \end{bmatrix}$
- $\Gamma_{ij} = \langle \overrightarrow{J}_i(t) \overline{\overrightarrow{J}_j(t)} \rangle$
- $\Gamma = \langle \overrightarrow{J(t)} \overline{\overrightarrow{J(t)}}^t \rangle$

Coherence Matrix: explicit formulation

$$\Gamma = \begin{bmatrix} \Gamma_{xx} & \Gamma_{xy} \\ \Gamma_{yx} & \Gamma_{yy} \end{bmatrix}$$

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$$\Gamma = \begin{bmatrix} \langle |A_x(t)|^2 \rangle & \langle A_x(t)\, \overline{A_y(t)} e^{i(\varphi_x - \varphi_y)} \rangle \\ \langle \overline{A_x(t)} A_y(t) e^{-i(\varphi_x - \varphi_y)} \rangle & \langle |A_y(t)|^2 \rangle \end{bmatrix}$$

Jones Coherence Matrix: properties

The Coherence Matrix

$$\Gamma = \begin{bmatrix} \left\langle \left| A_x(t) \right|^2 \right\rangle & \left\langle A_x(t)\, \overline{A_y(t)} e^{i(\varphi_x - \varphi_y)} \right\rangle \\ \left\langle \overline{A_x(t)} A_y(t) e^{-i(\varphi_x - \varphi_y)} \right\rangle & \left\langle \left| A_y(t) \right|^2 \right\rangle \end{bmatrix}$$

Trace is Intensity

$$\mathrm{Tr}(\Gamma) = I$$

Base change — Transformation $\mathbf{P}$

$$\mathbf{P}^{-1} \Gamma \mathbf{P}$$

Relationship with Stokes parameters — from definition

$$\begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & i & -i \end{bmatrix} \begin{bmatrix} \Gamma_{xx} \\ \Gamma_{yy} \\ \Gamma_{xy} \\ \Gamma_{yx} \end{bmatrix}$$

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Inverse relationship

$$\begin{bmatrix} \Gamma_{xx} \\ \Gamma_{yy} \\ \Gamma_{xy} \\ \Gamma_{yx} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -i \\ 0 & 0 & -1 & i \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

Coherence Matrix: further properties

Polarization degree

$$p = \sqrt{\frac{P_1^2+P_2^2+P_3^2}{P_0^2}} = \sqrt{1 - \frac{4(\Gamma_{xx}\Gamma_{yy}-\Gamma_{xy}\Gamma_{yx})}{(\Gamma_{xx}+\Gamma_{yy})^2}} = \sqrt{1 - \frac{4\mathrm{Det}(\Gamma)}{\mathrm{Tr}(\Gamma)^2}}$$

Γ Decomposition in polarized and depolarized components

- $\Gamma = \Gamma_P + \Gamma_{NP}$
- Find Γ_P and Γ_{NP} using the relationship with the Stokes parameters

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Propagation of the Coherence Matrix

Jones Calculus

- If incoming polarization is $\overrightarrow{J(t)}$
- Output one is $\overrightarrow{J'(t)} = \mathbf{M}\overrightarrow{J(t)}$

Coherence Matrix if M is unitary

- $\mathbf{M}$ unitary means : linear and/or circular anisotropy only
- $\Gamma' = \left\langle \overrightarrow{J'(t)}\,\overline{\overrightarrow{J'(t)}}^{t} \right\rangle$
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Propagating the Jones coherence matrix is difficult if the operator is not unitary

Jones Calculus raises some difficulties

- Coherence matrix OK for partially polarized light
- Propagation through unitary optical devices
  (linear or circular anisotropy only)
- Hard Times if Polarizers are present

The Stokes parameters may be an alternative

- Describing intensity, they can be readily measurered
- We will show they can be propagated using 4×4 real matrices
- They are the Mueller matrices

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The projection on a polarization state

Matrix of the polariazer with axis parallel to $\overrightarrow{V}$

$\overrightarrow{V}$

Projection on $\overrightarrow{V}$ in Jones Basis

$\mathbf{P_V}$

- Orthogonal Linear Polarizations Basis: $\overrightarrow{X}$ and $\overrightarrow{Y}$
- Normed Projection Base Vector :
  - $\overrightarrow{V} = A_x e^{-i\frac{\varphi}{2}} \overrightarrow{X} + A_y e^{i\frac{\varphi}{2}} \overrightarrow{Y}$
  - $\overline{\overrightarrow{V}}^{\mathrm{t}} \overrightarrow{V} = 1$
- $\mathbf{P_V} = \overrightarrow{V}\,\overline{\overrightarrow{V}}^{\mathrm{t}}$ [a]

---

[a] Easy to check in the projection eigen basis : $\left(\overrightarrow{V}, \overline{\overrightarrow{V}}^{\mathrm{t}}\right)$

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[a] Easy to check in the projection eigen basis : $\left(\overrightarrow{V}, \overline{\overrightarrow{V}}^{t}\right)$

The Pauli Matrices

A base for the 4D 2×2 matrix vector space

$$\sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \sigma_2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \sigma_3 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sigma_1 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$\mathbf{P_V}$ decomposition

$$\mathbf{P_V} = \frac{1}{2}\left(p_0\sigma_0 + p_1\sigma_1 + p_2\sigma_2 + p_3\sigma_3\right)$$

$\mathbf{P_V}$ composition and Trace property

Trace is the eigen values sum

Projection property

$$\overrightarrow{V}^{t} \cdot \sigma_{\mathbf{j}} \overrightarrow{V} = \left(\overrightarrow{V}^{t} \overrightarrow{V} \right) \overrightarrow{V}^{t} \cdot \sigma_{\mathbf{j}} \overrightarrow{V}$$

Projection Trace in its eigen basis

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$$\overline{\overrightarrow{V}}^{\mathrm{t}} \cdot \sigma_{\mathbf{j}} \overrightarrow{V} = \left(\overline{\overrightarrow{V}}^{\mathrm{t}} \overrightarrow{V} \right) \overline{\overrightarrow{V}}^{\mathrm{t}} \cdot \sigma_{\mathbf{j}} \overrightarrow{V} = \overline{\overrightarrow{V}}^{\mathrm{t}} \cdot \mathbf{P_V} \sigma_{\mathbf{j}} \overrightarrow{V}$$

Projection Trace in its eigen basis

- $\mathbf{P_V}$ eigenvalues : 0 & 1 $\qquad \mathrm{Tr}\left(P_V\right) = 1$
- $\mathbf{P_V}\sigma_{\mathbf{j}}$ eigenvalues : 0 & α $\qquad \alpha \leq 1 \qquad \mathrm{Tr}\left(P_V \sigma_j\right) = \alpha$
- $\mathbf{P_V}\sigma_{\mathbf{j}}$ eigenvectors are the same as $\mathbf{P_V}$:
  - $\overrightarrow{V}$ associated to eigenvalue α
- Project the projection

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$\mathbf{P_V}$ Pauli components and physical meaning

Express p_i as a function of $\overrightarrow{V}$ and the Pauli matrices, then find their signification

$\overrightarrow{V}^{\mathrm{t}} \cdot \sigma_{\mathbf{j}} \overrightarrow{V} = \mathrm{Tr}\left(P_V \sigma_j\right)$ $\qquad$ $\mathrm{Tr}\left(\sigma_i \sigma_j\right) = 2\delta_{ij}$

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Project the base vectors on $\overrightarrow{V}$

- Using $\overrightarrow{V} = A_x e^{-i\frac{\varphi}{2}} \overrightarrow{X} + A_y e^{i\frac{\varphi}{2}} \overrightarrow{Y}$
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$\mathbf{P_V}$ Pauli composition and Stokes parameters

Stokes parameters as $\mathbf{P_V}$ decomposition on the Pauli base

- $p_0 = P_0 = A_x^2 - A_y^2 = I_x - I_y$
- $p_1 = P_1 = A_x^2 - A_y^2 = I_x - I_y$
- $p_2 = P_2 = 2A_xA_y\cos(\varphi) = I_{\pi/4} - I_{-\pi/4}$
- $p_3 = P_3 = 2A_xA_y\sin(\varphi) = I_\mathrm{L} - I_\mathrm{R}$

Propagating through devices: Mueller matrices

$\overrightarrow{V'} = \mathbf{M_J}\,\overrightarrow{V}$

Projection on $\overrightarrow{V'}$

$$\mathbf{P_{V'}} = \overrightarrow{V'}\,\overline{\overrightarrow{V'}}^{\mathrm{t}}$$

Trace relationship

$$P'_i = \mathrm{Tr}\left(\mathbf{P_{V'}}\sigma_i\right)$$

Mueller matrix

$\overrightarrow{S'} = \mathbf{M_M}\,\overrightarrow{S}$

$$\left(M_M\right)_{ij} = \frac{1}{2}\mathrm{Tr}\left(\mathbf{M_J}\sigma_{\mathbf{j}}\overline{\mathbf{M_J}}^{\mathrm{t}}\sigma_{\mathbf{i}}\right)$$

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Mueller matrices and partially polarized light

Time average of the previous study

Mueller matrices are time independent

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Mueller calculus can be extended to...

- Partially coherent light
- Cascaded optical devices

Final homework

Find the Mueller matrix of each :

- Polarizers along eigen axis or θ tilted
- half and quarter wave plates
- linearly and circularly birefringent crystal

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