Optical Phase Conjugation
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Principle and application of phase conjugation

Generation of phase conjugate waves

Self Pumping and Holography

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Optical Phase Conjugation

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Download this document: http://moodle.univ-metz.fr/
Useful reading... [YY84, Yar97, San99]

F. Sanchez.  
*Optique non-linéaire - Cours et problèmes résolus.*  
Ellipses, 1999.

A. Yariv. 
*Optical Electronics in Modern Communications.*  

A. Yariv and P. Yeh.  
*Optical waves in crystals. Propagation and control of laser radiation.*  

... and many others.
Contents

1 Principle and application of phase conjugation
   - Experiment
   - Distortion correction theorem

2 Generation of phase conjugate waves
   - Non Linear Polarization Development
   - Four wave mixing coupled mode formulation
   - Conjugate wave amplitude

3 Self Pumped Phase Conjugation and Holography
   - Four Wave Mixing from a holographic point of view
   - Phase Conjugation without pumping
A peculiar phenomenon
Discovered in the early 70\textsuperscript{ies}

Use Non Linear Material

- Third order: non zero $\chi_3$
  - Photorefractivity
  - Stimulated Brillouin Scattering
  - Stimulated Raman Scattering
- Four wave mixing

Wavefront correction

- Distorted incident beam
- Reflected back \textbf{as is}
- Distortion corrected

\textbf{Figure:} Phase conjugation principle.
Principle and application of phase conjugation

Experiment

**Beams are reflected back *as if time was reversed***

Images source:
http://sharp.bu.edu/~slehar/PhaseConjugate/PhaseConjugate.html

Incident wavefronts...

- Are reflected back **exactly**
- Back and forth wavefronts are **identical**
Attractive applications
All based on wavefront distortion correction

Phase conjugation applications

- All optical image transmission through fibers
- Distortion correction in high power lasers
- Dynamic wave front correction for optical sensors
- Dynamic Holography
- ...
A phase conjugate wave travels time the wrong way
Phase conjugation is also known as *time reversal*

Take some input monochromatic wave

\[ E_1 = \text{Re} \left[ \psi(r) \exp(i(\omega t - kz)) \right] = \text{Re} \left[ \psi(r) \right] \cos(\omega t - kz) \]

Take the phase conjugate *over space only*

\[ E_2 = \text{Re} \left[ \overline{\psi(r)} \exp(i(-kz))e^{i\omega t} \right] \]
\[ E_2 = \text{Re} \left[ \overline{\psi(r)} \exp(i(\omega t + kz)) \right] \]
\[ E_2 = \text{Re} \left[ \overline{\psi(r)} \right] \cos(\omega t + kz) \]
\[ E_2 = \text{Re} \left[ \overline{\psi(r)} \right] \cos(-\omega t - kz) \]
The distortion correction theorem

If a backward traveling wave is phase conjugate *somewhere* then it is *everywhere*

---

**Take a paraxial forward propagating wave**

- Expressed as: \( E_1 (r, t) = \psi_1 (r) e^{i(\omega t - kz)} \)
- Obeys the wave equation: \( \Delta E_1 + \omega^2 \mu_0 \varepsilon (r) E_1 = 0 \)
- In the paraxial limit: \( \Delta \psi_1 + \left[ \omega^2 \mu_0 \varepsilon (r) - k^2 \right] \psi_1 - 2ik \frac{\partial \psi_1}{\partial z} = 0 \)
- Conjugate equation: \( \Delta \overline{\psi_1} + \left[ \omega^2 \mu_0 \varepsilon (r) - k^2 \right] \overline{\psi_1} + 2ik \frac{\partial \overline{\psi_1}}{\partial z} = 0 \)

**Had we taken a backward propagating wave**

- Expressed as: \( E_2 (r, t) = \psi_2 (r) e^{i(\omega t + kz)} \)
- Paraxial equation \((z \to -z)\): \( \Delta \psi_2 + \left[ \omega^2 \mu_0 \varepsilon (r) - k^2 \right] \psi_2 + 2ik \frac{\partial \psi_2}{\partial z} = 0 \)

**Same second order linear differential equations for loss-less media**

\( \varepsilon (r) \in \mathbb{R} \Rightarrow \left[ \psi_2 (z = 0) = a.\overline{\psi_1 (z = 0)} \iff \forall z, \psi_2 (z) = a.\overline{\psi_1 (z)} \right] \)
**Flashback : Second Order**

- Relies on $\chi_2 : P_{NL} \propto E^2$
- Two waves mix to generate a third one
  \[ \omega_1 \pm \omega_2 \rightarrow \omega_3 \]
- Second Harmonic Generation, Optical Parametric Amplification, Optical Parametric Oscillation...

**Third order**

- Relies on $\chi_3 : P_{NL} \propto E^3$
- Three waves mix to generate a fourth one
  \[ \omega_1 \pm \omega_2 \pm \omega_3 \rightarrow \omega_4 \]
- Phase conjugation for $\omega_4 = \omega_1 + \omega_2 - \omega_3$? Let’s see…
Non Linear Polarization Development

Non Linear Polarization \(P_{NL}\)

General polarization development

\[
[P]_i = \varepsilon_0[\chi]_{ij}[E]_j + 2[d]_{ijk}[E]_j[E]_k + 4[\chi]_{ijkl}[E]_j[E]_k[E]_l
\]

Third order non linear development\(^1\) around \(\omega_4 = \omega_1 + \omega_2 - \omega_3\)

\[
[P_{NL}]_i(\omega_4) = 6[\chi]_{ijkl}[E]_j(\omega_1)[E]_k(\omega_2)[E]_l(\omega_3)e^{i(\omega t + k_4 r)}
\]

Degenerate Four wave mixing: \(\omega = \omega + \omega - \omega\)

\[
[P_{NL}]_i(\omega) = 6[\chi]_{ijkl}[E]_j(\omega)[E]_k(\omega)[E]_l(\omega)e^{i(\omega t + kr)}
\]

\(^1\)As an exercise, you can multiply, sum-up and keep only \(\omega_4\) related terms... and find \(k_4\)
Degenerate Four Wave mixing configuration

- $A_1 = \overline{A}_2$ intense plane pumps
- $A_3$ is the signal
- We seek $A_4$
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Four wave mixing coupled mode formulation

Signal wave equation

Let us start with the standard non linear wave equation

\[ \Delta E - \mu_0 \varepsilon \frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial^2 P_{NL}}{\partial t^2} \]

Signal \( A_3 \) propagation

- Each wave has its own direction and polarization
- They can be treated separately
- With \( \Delta E_3 = \Re \left[ \left( -k^2 A_3 - 2ik \frac{\partial A_3}{\partial z} + \frac{\partial^2 A_3}{\partial z^2} \right) e^{i(\omega t-kz)} \right] \)
- \( \Re \left[ \left( -2ik \frac{\partial A_3}{\partial z} + \frac{\partial^2 A_3}{\partial z^2} \right) e^{i(\omega t-kz)} \right] = \mu_0 \frac{\partial^2 P_{NL}}{\partial t^2} \)

Evaluation of the non linear polarization \( P_{NL} \)

Let us take a look at the terms which involve \( e^{i(\omega t \pm kz)} \)
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**Stripping the non linear polarization to useful terms**

Keeping only the relevant terms which contain $e^{i(\omega t \pm kz)}$

Expansion of third order non linear polarization

$$[P_{NL}]_i = \Re e \left[ 6 \left( \begin{array}{c} [\chi]_{ijkl} [A_1]_j [A_2]_k [A_4]_l \\ + [\chi]_{iji} [A_1]_j [A_1]_j [A_3]_i \\ + [\chi]_{ikki} [A_2]_k [A_2]_k [A_3]_i \end{array} \right) e^{i(\omega t - kz)} \right]$$

Simplifying assumptions

- Intense pump beam terms are dominant
- Polarizations are
  - either all the same, only $[\chi]_{iii}$ involved
  - or $(A_1/A_2) \perp (A_3/A_4)$, only $[\chi]_{iji}$, $i \neq j$ involved

$$[P_{NL}]_i = \chi^{(3)} \left( (||A1||^2 + ||A2||^2) A_3 + A_1 A_2 A_4 \right) e^{i(\omega t - kz)}$$

$$\chi^{(3)} = 6[\chi]_{iii} \text{ or } \chi^{(3)} = 6[\chi]_{iji}$$
Resulting coupled wave propagation equation

Coupled wave equation resulting of $P_{NL}$

$$\frac{\partial A_3}{\partial z} = -i \frac{\omega}{2} \sqrt{\frac{\mu_0}{\varepsilon}} \chi^{(3)} \left[ (\|A_1\|^2 + \|A_2\|^2) A_3 + A_1 A_2 A_4 \right]$$

Further simplification

- Homogeneous refraction index modulation: Kerr effect
  - Simple phase factor change
  - Remove it from equation $A'_i = A_i e^{-i \frac{\omega}{2} \sqrt{\frac{\mu_0}{\varepsilon}} \chi^{(3)} (\|A_1\|^2 + \|A_2\|^2)z}$
- Set $\kappa = \frac{\omega}{2} \sqrt{\frac{\mu_0}{\varepsilon}} \chi^{(3)} A_1 A_2$

Simplified coupling equations

$$\frac{\partial A'_3}{\partial z} = i \kappa A'_4$$
$$\frac{\partial A'_4}{\partial z} = i \kappa A'_3$$

obtained through the same kind of derivation
**General solution**

Boundary conditions at $z = 0$ and $z = L$

<table>
<thead>
<tr>
<th>$A_3$ is forward propagating</th>
<th>$A_4$ is backward propagating</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A'_3(z) = -i\frac{</td>
<td>\kappa</td>
</tr>
</tbody>
</table>

**One beam experiment**

$A'_4(L) = 0$

| $A'_3(L) = \frac{A'_3(0)}{\cos(|k| L)}$ | Coherent amplifier |
| $A'_4(0) = -i \frac{|\kappa|}{|\kappa|} \tan(|\kappa| L) \overline{A'_3(0)}$ | Reflectivity can exceed 1 |
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One Beam experiment and phase factor

One beam experiment

\[ A'_3 (L) = \frac{A'_3(0)}{\cos(|k| L)} \]

\[ A'_4 (0) = -i \frac{\kappa}{|\kappa|} \tan (|\kappa| L) \overline{A'_3 (0)} \]

Coherent amplifier

Reflectivity can exceed 1

What if \( \cos (|k| L) = 0 \)?

- Infinite gain
- \( A_3 \) and \( A_4 \) start from noise
- Spontaneous oscillations
Four Wave Mixing from a holographic point of view

Four Wave Mixing is Real Time Holography

**Write Hologram**

\[ T \propto \|A_1 + A_3\|^2 = \|A_1\|^2 + \|A_3\|^2 + \overline{A_1A_3} + \overline{A_3A_1} \]

**Read Hologram with \(A_2 = \overline{A_1}\)**

\[ A_4 \propto TA_2 = (\|A_1\|^2 + \|A_3\|^2)A_2 + A_2\overline{A_1A_3} + A_2\overline{A_3A_1} \]
CAT Conjugator using Beam Fanning

Figure: Beam fanning in photorefractive Baryum Titanate