Optical Phase Conjugation

Nicolas Fressengeas

UE SPM-PHO-S09-112
Optical Phase Conjugation

N. Fressengeas

Laboratoire Matériaux Optiques, Photonique et Systèmes
Unité de Recherche commune à l'Université Paul Verlaine Metz et à Supélec

Dowload this document : `http://moodle.univ-metz.fr/`

Usefull reading. . .

[YY84, Yar97, San99]

F. Sanchez.
Optique non-linéaire - Cours et problèmes résolus.
Ellipses, 1999.

A. Yariv.
Optical Electronics in Modern Communications.
Oxford Series in Electrical and Computer Engineering. Oxford University Press, 1997.

A. Yariv and P. Yeh.
Optical waves in crystals. Propagation and control of laser radiation.
Wiley series in pure and applied optics. Wiley-Interscience, Stanford University, 1984.

. . . and many others.

Contents

1. Principle and application of phase conjugation
   - Experiment
   - Distortion correction theorem
2. Generation of phase conjugate waves
   - Non Linear Polarization Development
   - Four wave mixing coupled mode formulation
   - Conjugate wave amplitude
3. Self Pumped Phase Conjugation and Holography
   - Four Wave Mixing from a holographic point of view
   - Phase Conjugation without pumping

A peculiar phenomenon
Discovered in the early 70^{ies}

Use Non Linear Material

- Third order : non zero χ_3
  - Photorefractivity
  - Stimulated Brillouin Scattering
  - Stimulated Raman Scattering
- Four wave mixing

Wavefront correction

- Distorted incident beam
- Reflected back as is
- Distortion corrected

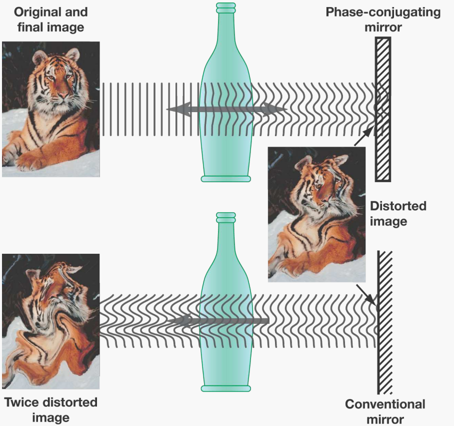


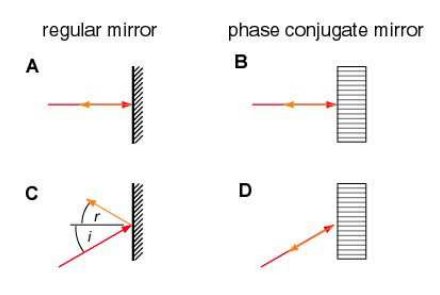
Figure: Phase conjugation principle.
Source : Wikipedia

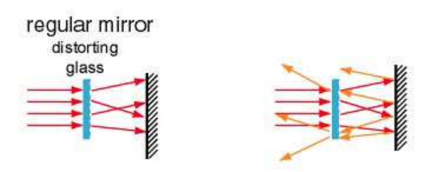
Beams are reflected back *as if time was reversed*

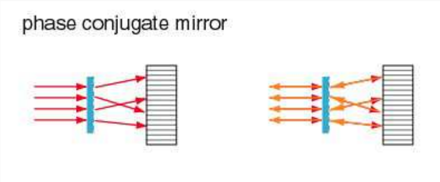
Images source :
http://sharp.bu.edu/~slehar/PhaseConjugate/PhaseConjugate.html

Incident wavefronts. . .

- Are reflected back exactly
- Back and forth wavefronts are identical







Attractive applications

All based on wavefront distortion correction

Phase conjugation applications

- All optical image transmission through fibers
- Distortion correction in high power lasers
- Dynamic wave front correction for optical sensors
- Dynamic Holography
- ...

A phase conjugate wave travels time the wrong way

Phase conjugation is also known as *time reversal*

Take some input monochromatic wave

$$E_1 = \mathcal{R}e\left[\psi(r)\exp\left(i\left(\omega t - kz\right)\right)\right] = \mathcal{R}e\left[\psi(r)\right]\cos\left(\omega t - kz\right)$$

Take the phase conjugate *over space only*

- $E_2 = \mathcal{R}e\left[\overline{\psi(r)\exp\left(i\left(-kz\right)\right)}e^{i\omega t}\right]$
- $E_2 = \mathcal{R}e\left[\overline{\psi(r)}\exp\left(i\left(\omega t + kz\right)\right)\right]$
- $E_2 = \mathcal{R}e\left[\overline{\psi(r)}\right]\cos\left(\omega t + kz\right)$
- $E_2 = \mathcal{R}e\left[\psi(r)\right]\cos\left(-\omega t - kz\right)$

The distortion correction theorem

If a backward traveling wave is phase conjugate *somewhere* then it is *everywhere*

Take a paraxial forward propagating wave

- Expressed as: $E_1(r,t) = \psi_1(r)\, e^{i(\omega t - kz)}$
- Obeys the wave equation: $\Delta E_1 + \omega^2 \mu_0 \varepsilon(r) E_1 = 0$
- In the paraxial limit: $\Delta \psi_1 + \left[\omega^2 \mu_0 \varepsilon(r) - k^2\right] \psi_1 - 2ik \frac{\partial \psi_1}{\partial z} = 0$
- Conjugate equation: $\Delta \overline{\psi_1} + \left[\omega^2 \mu_0 \overline{\varepsilon(r)} - k^2\right] \overline{\psi_1} + 2ik \frac{\partial \overline{\psi_1}}{\partial z} = 0$

Had we taken a backward propagating wave

- Expressed as: $E_2(r,t) = \psi_2(r)\, e^{i(\omega t + kz)}$
- Paraxial equation ($z \to -z$):
  $\Delta \psi_2 + \left[\omega^2 \mu_0 \varepsilon(r) - k^2\right] \psi_2 + 2ik \frac{\partial \psi_2}{\partial z} = 0$

Same second order linear differential equations for loss-less media

$$\varepsilon(r) \in \mathbb{R} \Rightarrow \left[\psi_2(z=0) = a.\overline{\psi_1(z=0)} \Leftrightarrow \forall z, \psi_2(z) = a.\overline{\psi_1(z)}\right]$$

Four wave mixing
Third Order Non Linear Optics

Flashback : Second Order

- Relies on χ_2 : $P_{NL} \propto E^2$
- Two waves mix to generate a third one
- $\omega_1 \pm \omega_2 \longrightarrow \omega_3$
- Second Harmonic Generation, Optical Parametric Amplification, Optical Parametric Oscillation. . .

Third order

- Relies on χ_3 : $P_{NL} \propto E^3$
- Three waves mix to generate a fourth one
- $\omega_1 \pm \omega_2 \pm \omega_3 \longrightarrow \omega_4$
- Phase conjugation for $\omega_4 = \omega_1 + \omega_2 - \omega_3$? Let's see. . .

paul verlaine université - metz

Non Linear Polarization P_{NL}

General polarization development

$$[P]_i = \varepsilon_0[\chi]_{ij} + 2[d]_{ijk}[E]_j[E]_k + 4[\chi]_{ijkl}[E]_j[E]_k[E]_l$$

Third order non linear development[1] around $\omega_4 = \omega_1 + \omega_2 - \omega_3$

$$[P_{NL}]_i\,(\omega_4) = 6[\chi]_{ijkl}[E]_j\,(\omega_1)\,[E]_k\,(\omega_2)\,\overline{[E]_l\,(\omega_3)}e^{i(\omega_4 t + k_4 r)}$$

Degenerate Four wave mixing : $\omega = \omega + \omega - \omega$

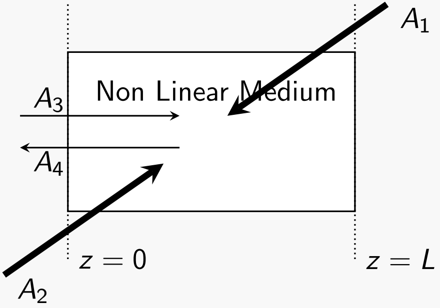
$$[P_{NL}]_i\,(\omega) = 6[\chi]_{ijkl}[E]_j\,(\omega)\,[E]_k\,(\omega)\,\overline{[E]_l\,(\omega)}e^{i(\omega t + kr)}$$

[1] As an exercise, you can multiply, sum-up and keep only ω_4 related terms. . . and find k_4

Degenerate Four Wave mixing configuration

Degenerate configuration

- $A_1 = \overline{A_2}$ intense plane pumps
- A_3 is the signal
- We seek A_4



Signal wave equation

Let us start with the standard non linear wave equation

$$\Delta E - \mu_0 \varepsilon \frac{\partial^2 E}{\partial t^2} = \mu_0 \frac{\partial^2 P_{NL}}{\partial t^2}$$

Signal A_3 propagation

- Each wave has its own direction and polarization
- They can be treated separately
- With $\Delta E_3 = \mathcal{R}e\left[\left(-k^2 A_3 - 2ik\frac{\partial A_3}{\partial z} + \frac{\partial^2 A_3}{\partial z^2}\right) e^{i(\omega t - kz)}\right]$
- $\mathcal{R}e\left[\left(-2ik\frac{\partial A_3}{\partial z} + \frac{\partial^2 A_3}{\partial z^2}\right) e^{i(\omega t - kz)}\right] = \mu_0 \frac{\partial^2 P_{NL}}{\partial t^2}$

Evaluation of the non linear polarization P_{NL}

Let us take a look at the terms which involve $e^{i(\omega t \pm kz)}$

Stripping the non linear polarization to useful terms

Keeping only the relevant terms which contain $e^{i(\omega t \pm kz)}$

Expansion of third order non linear polarization

$$[P_{NL}]_i = \mathcal{R}e \left[6 \begin{pmatrix} & [\chi]_{ijkl} & [A_1]_j & [A_2]_k & \overline{[A_4]_l} \\ + & [\chi]_{ijji} & [A_1]_j & \overline{[A_1]_j} & \overline{[A_3]_i} \\ + & [\chi]_{ikki} & [A_2]_k & \overline{[A_2]_k} & [A_3]_i \end{pmatrix} e^{i(\omega t - kz)} \right]$$

Simplifying assumptions

- Intense pump beam terms are dominant
- Polarizations are
  - either all the same, only $[\chi]_{iiii}$ involved
  - or $(A_1/A_2) \perp (A_3/A_4)$, only $[\chi]_{ijji}, i \neq j$ involved
  - $\chi^{(3)} = 6[\chi]_{iiii}$ or $\chi^{(3)} = 6[\chi]_{ijji}$

Resulting coupled wave propagation equation

Coupled wave equation resulting of P_{NL}

$$\frac{\partial A_3}{\partial z} = -i\frac{\omega}{2}\sqrt{\frac{\mu_0}{\varepsilon}}\chi^{(3)}\left[\left(\|A1\|^2 + \|A2\|^2\right)A_3 + A_1 A_2 \overline{A_4}\right]$$

Further simplification

- Homogeneous refraction index modulation : Kerr effect
  - Simple phase factor change
  - Remove it from equation $A'_i = A_i e^{-i\frac{\omega}{2}\sqrt{\frac{\mu_0}{\varepsilon}}\chi^{(3)}\left(\|A_1\|^2+\|A_2\|^2\right)z}$
- Set $\overline{\kappa} = \frac{\omega}{2}\sqrt{\frac{\mu_0}{\varepsilon}}\chi^{(3)}A_1 A_2$

Simplified coupling equations

- $\frac{\partial \overline{A'_3}}{\partial z} = i\kappa A'_4$
- $\frac{\partial A'_4}{\partial z} = i\overline{\kappa A'_3}$ obtained through the same kind of derivation

General solution

Boundary conditions at $z = 0$ and $z = L$

A_3 is forward propagating **A_4 is backward propagating**

- $A_3'(z) = -i\frac{|\kappa|\sin(|k|\,z)}{\kappa\cos(|k|\,L)}\overline{A_4'(L)} + \frac{\cos(|k|\,(z-L))}{\cos(|k|\,L)}A_3'(0)$
- $A_4'(z) = \frac{\cos(|k|\,z)}{\cos(|k|\,L)}A_4'(L) + i\frac{\overline{\kappa}\sin(|k|\,(z-L))}{|k|\cos(|k|\,L)}\overline{A_3'(0)}$

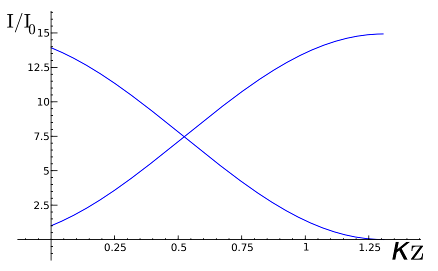
One beam experiment **$A_4'(L) = 0$**

- $A_3'(L) = \frac{A_3'(0)}{cos(|k|L)}$ — Coherent amplifier
- $A_4'(0) = -i\frac{\overline{\kappa}}{|\kappa|}\tan(|\kappa|\,L)\,\overline{A_3'(0)}$ — Reflectivity can exceed 1

One Beam experiment and phase factor

One beam experiment $A_4'(L) = 0$

- $A_3'(L) = \frac{A_3'(0)}{\cos(|k|L)}$ Coherent amplifier
- $A_4'(0) = -i\frac{\overline{\kappa}}{|\kappa|}\tan(|\kappa|\, L)\,\overline{A_3'(0)}$ Reflectivity can exceed 1



What if $\cos(|k|\, L) = 0$?

- Infinite gain
- A_3 and A_4 start from noise
- Spontaneous oscillations

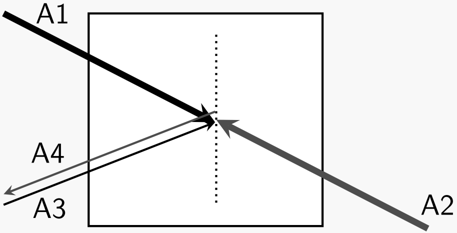
Four Wave Mixing is Real Time Holography

Write Hologram

$$T \propto \|A_1 + A_3\|^2 = \|A_1\|^2 + \|A_3\|^2 + A_1\overline{A_3} + A_3\overline{A_1}$$

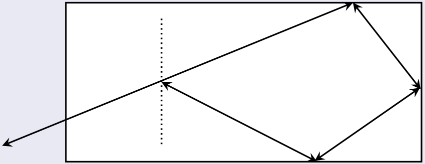
Read Hologram with $A_2 = \overline{A_1}$

$$A_4 \propto TA_2 = \left(\|A_1\|^2 + \|A_3\|^2\right) A_2 + A_2 A_1 \overline{A_3} + A_2 A_3 \overline{A_1}$$



CAT Conjugator using Beam Fanning

Total internal reflection



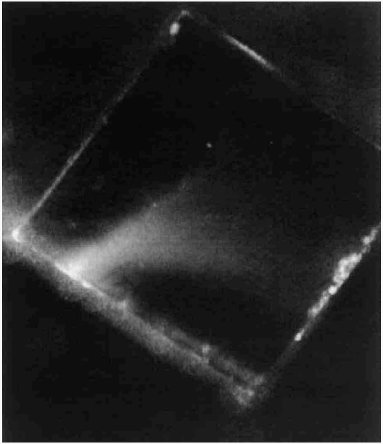


Figure: Beam fanning in photorefractive Baryum Titanate