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Photorefractivity

N. Fressengeas

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Unité de Recherche commune à l’Université Paul Verlaine Metz et à Supélec

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Photorefractive effect
History began in 1966 as *Optical Damage*

### Optical Damage in LiNbO₃

- Shine a light on LiNbO₃
- Remove it
- Shine another: damaged crystal

### Semi-permanent effect

- Leave it in the dark: still damaged
- Leave it under uniform light: sometimes repaired

### Today

- Photorefractivity can prove useful
- Some people still call it *optical damage*:
  - Bad for linear optics (electro-optic modulators…)
  - Bad for *instant* Non Linear Optics (SHG, OPA…)

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**Photorefractivity**
Version 1.2
frame 2

N. Fressengeas

Band Transport
Harmonic illumination
Two Wave Mixing
Photorefractive effect
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Photorefractivity is attractive
Non linear optics at low optical power

Non Linear Optics

- Dynamic Holography
- Phase conjugation
- All optical computing
- ...
- At milliwatts and below power levels

Observed in many non linear crystals

- Sillenites: Bi$_{12}$SiO$_{20}$, Bi$_{12}$TiO$_{20}$, Bi$_{12}$GeO$_{20}$
- Tungsten-Bronze: Sr$_x$Ba$_{1-x}$Nb$_2$O$_6$
- Ferroelectrics: LiNbO$_3$, BaTiO$_3$
- Semiconductors: InP:Fe, AsGa
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Usefull reading...

[Yeh93, GH88, GH89]

P. Günter and J. P. Huignard.  
*Photorefractive materials and their applications I*, volume 61 of *Topics in Applied Physics*.  

P. Günter and J. P. Huignard.  
*Photorefractive materials and their applications II*, volume 62 of *Topics in Applied Physics*.  

P. Yeh.  
*Introduction to photorefractive nonlinear optics*.  
Contents

1 Band Transport Model
   - Schematics
   - Carrier Generation
   - Charge Transport
   - Electro-optic effect

2 Harmonic illumination
   - Harmonic framework
   - Uniform background: order 0
   - Periodic modulation: order 1
   - Implications, Simplifications, Diffusion and Saturation

3 Two Wave Mixing
   - Gratings graphical view
   - Coupled waves
   - Two Beam Coupling
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Photorefractive charge transport and trapping

Linear Index Modulation

Space charge electric field generates refractive index variation through electro-optic effect
Photorefractive charge transport and trapping

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Periodic illumination of a photorefractive material

- Optical Intensity
- Charge density
- Electric Field
- Refractive Index Change
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Donors wanted
Carriers are generated by donors: no donors, no carriers

Nominally pure crystals

- No in-band-gap level
  - No donor nor acceptor
  - No photorefractive effect
- Structural defects often present
- As well as pollutants
- They create in-band-gap levels
- Photorefractivity can arise from them

More efficient: doping

- Introduce in-band-gap species
- LiNbO$_3$:Fe, InP:Fe...
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Introduce Donors of electrons $N_D$
- Energy level close to conduction band
- They easily **give electrons** to conduction band

Introducing Acceptors $N_A \ll N_D$
- Photorefractivity needs traps
- Ionized donors are traps
- Introduce Acceptors close to the valence band
- They catch Donors electrons
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Thermal carrier generation

We assume here that the only carriers are electrons... what if not?

Comes from temperature induced Brownian motion

- Temperature induced
- Electrons are **kicked** into conduction band

Rate proportional to donors-left-to-ionize density

\[
\frac{\partial n_e}{\partial t} = \beta (N_D - N_D^+) 
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One generated electron leaves one ionized donor

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Photo-excitation of carriers

The photoelectric effect at work

- Photon **energy sufficient** to reach conduction band
- Rate proportional to light intensity $I$
- And to left-to-ionize donors

Photo-excitation rate

$$\frac{\partial n_e}{\partial t} = \frac{\partial N_D^+}{\partial t} = \sigma I (N_D - N_D^+)$$

The photo-ionization cross section $\sigma$

- Has the dimensions of a surface
- If $I$ is given as a number of photon per surface units and time
- Sometimes the case, sometimes not...
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Carrier recombination

Recombination needs luck, electrons and empty traps

- A luck factor \( \xi \)
- Carriers density \( n_\text{e} \)
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Carriers rate equation
A combination of generation and recombination

\[
\frac{\partial N_D^+}{\partial t} = (\beta + \sigma I) (N_D - N_D^+) - \xi n_e N_D^+
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Charge transport in the conduction band
An assumption of the Band Transport Model... sometimes untrue

**Diffusion**
- Due to Temperature and Brownian motion
- Think of it as *sugar in water* (or coffee)
- Depends on concentration variations

**Drift under electric-field**
- Needs electric-field
- Externally applied or diffusion generated
- Depends on electric field and mobility

**Photovoltaic effect**
- Sometimes called *photo-galvanic*
- Non-isotropic effect
- Think of solar cells: light generates current
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Diffusion transport
Diffusion current from Fick’s first law linked to Einstein relation

**Fick’s first law**

\[ \mathbf{J}_p = -D \nabla p \]

**Einstein relation**

- Links diffusion, absolute temperature \( T \) and Brownian motion through mobility
- Mobility \( \mu \) is the velocity to electric field ratio
  \[ D = \mu_e k_B T / e \]

**Diffusion Current**

\[ \mathbf{J} = -e \mathbf{J}_e \]
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A charged particle in an electric field...

Electric Field

- Externally applied
- Due to charged carrier diffusion

Drift current

- Electrons velocity: \( \mathbf{v} = -\mu_e \mathbf{E} \)
- Drift Current: \( \mathbf{J} = -e \times \mathbf{v} \)
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Photovoltaic current
An non isotropic effect stemming from crystal asymmetry

Origins
- Non centro-symmetric crystal e.g. LiNbO₃
- Anisotropic photo-electric effect
- Depends on light polarization

Photovoltaic tensor
- Rank 2
- Main component along polar axis
- Often reduced to a scalar

Photovoltaic Current
\[
\mathbf{J}_i = (N_D - N_D^+) \sum_{j,k} \beta^{ph}_{j,k} \mathbf{E}_j \mathbf{E}_k \mathbf{u}_i
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\mathbf{J} &\approx \beta^{ph} I (N_D - N_D^+) \mathbf{c}
\end{align*}
\]
Photovoltaic current
An non isotropic effect stemming from crystal asymmetry

**Origins**
- Non centro-symmetric crystal
  - e.g. LiNbO$_3$
- Anisotropic photo-electric effect
- Depends on light polarization

**Photovoltaic tensor**
- Rank 2
- Main component along polar axis
- Often reduced to a scalar

**Photovoltaic Current**

\[
\left[ \vec{J} \right]_i = (N_D - N_D^+) \sum_{j,k} [\beta^{ph}]_{j,k} \left[ \vec{E} \right]_j \left[ \vec{E} \right]_k \vec{u}_i
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\[
\vec{J} \approx \beta^{ph} I (N_D - N_D^+) \vec{c}
\]
Band Transport Model
Also known as Kukhtarev model
Published in 1979

Ionized donors rate equation

\[ \frac{\partial N_D^+}{\partial t} = (\beta + \sigma \mathcal{I}) (N_D - N_D^+) - \xi n_\theta N_D^+ \]

Current density expression

\[ \mathbf{J} = \mu_0 k_B T \text{grad} (n_\theta) + en_\theta \mu_0 \mathbf{E} + \beta \varphi \mathcal{I} (N_D - N_D^+) \mathbf{c} \]

Quasi-static Maxwell model

- Continuity: \( \text{div} \left( \mathbf{J} \right) + \frac{\partial \rho}{\partial t} = 0 \)
- Charge: \( \rho = e (N_D^+ - N_A^- - n_\theta) \)
- Maxwell-Gauss: \( \text{div} \left( \mathbf{D} \right) = \rho \), with \( \mathbf{D} = \varepsilon \mathbf{E} \)
Ionized donors rate equation

\[
\frac{\partial N_D^+}{\partial t} = (\beta + \sigma I) (N_D - N_D^+) - \xi n_e N_D^+
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   - Carrier Generation
   - Charge Transport
   - Electro-optic effect

2 **Harmonic illumination**
   - Harmonic framework
   - Uniform background: order 0
   - Periodic modulation: order 1
   - Implications, Simplifications, Diffusion and Saturation

3 **Two Wave Mixing**
   - Gratings graphical view
   - Coupled waves
   - Two Beam Coupling
Refractive index modulation through electro-optics
Space-charge electric field induces refractive index variations

This is not an electro-optics lesson
Please refer to the electro-optics lesson

Anyhow...

- Light generated electric field: the space charge field $\vec{E}$
- In electro-optic materials: creates index modulation

$$[\Delta \frac{1}{n^2}]_{i,j} = \sum_k [r]_{ijk} \left[ \vec{E} \right]_k$$

- Local modulation of refractive index
- Local modification of refractive index ellipsoid
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Periodic illumination from plane waves interference

**Two plane waves interfering**

- Same wavelength and coherent
- non collinear wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$: $\mathbf{K} = \mathbf{k}_2 - \mathbf{k}_1$
- Interference pattern: $I_{(0)}(1 + m \cos (\mathbf{K} \cdot \mathbf{r}))$
  - $I_{(0)} = I_1 + I_2$
  - $m = 2 \frac{\sqrt{I_1 I_2}}{I_1 + I_2}$

**Harmonic assumptions**

- $m \ll 1$: intensities are very different
- All unknowns are sum of
  - A large uniform background: order 0
  - A small harmonic modulation: order 1
- Linearity: orders can be uncoupled
  - Uniform intensity analysis
  - Followed by small signal analysis
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Simplifying assumptions

One dimension problem 1D
- Plane waves interference
- All phenomena are collinear to $\vec{K}$

Drift-diffusion transport only assumed
Photovoltaic effect assumed negligible
Photo-generation only assumed
Large intensities: thermal generation can be neglected
Steady state study
All temporal derivatives assumed zero
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Carrier generation–recombination equilibrium

Steady state equilibrium

\[ \frac{\partial N_D^+}{\partial t} = \sigma I (N_D - N_D^+) - \xi n_e N_D^+ \]

Uniform electric field

- \( \overrightarrow{D}(0) \) is homogeneous

small illumination

- \( n_e \ll N_A \)
- \( \sigma I \ll \xi N_A \)

Equilibrium homogeneous densities

- \( N_D^+(0) = N_A + n_e \)
- \( n_e(0) = \frac{N_D^+(0) - N_D}{\sigma I} \)
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Order 1 framework
Basic multi-scale modeling

All quantities are assumed periodic

\[
\text{div}(\vec{X}) = i\vec{K} \cdot \vec{X}
\]

... 

Order 0 assumed known
Order 1 assumed small

- \( \forall \vec{X}, \, X_{(1)} \ll X_{(0)} \)
- \((\vec{X} \times \vec{Y})_{(1)} = X_{(0)} Y_{(0)} + X_{(0)} Y_{(1)} + X_{(1)} Y_{(0)} + X_{(1)} Y_{(1)}\)
- Order 0 is independently found
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Order one charge and current equilibrium

Steady state equilibrium

- \( \sigma I \left( N_D - N_D^+ \right) = \xi n_\varepsilon N_D^+ \)

- \( \sigma I_{(1)} \left( N_D - N_D^{+(0)} \right) + \sigma I_{(0)} \left( -N_D^{+(1)} \right) = \xi n_\varepsilon^{(0)} N_D^{+(1)} + \xi n_\varepsilon^{(1)} N_D^{+(0)} \)

Harmonic Current density

\[ \vec{J}_{(1)} = \mu_\varepsilon k_B T \text{grad} (n_\varepsilon)_{(1)} + e \mu_\varepsilon n_\varepsilon^{(1)} \vec{E}_{(1)} \]

Harmonic Current density divergence is null

\[ i \vec{K} \cdot \vec{J} = 0 \]

\[ i \vec{K} \cdot \left( \mu_\varepsilon k_B T n_\varepsilon^{(1)} \vec{K} + e \mu_\varepsilon n_\varepsilon^{(1)} \vec{E}_{(1)} \right) = 0 \]

Harmonic Poisson

\[ \text{div} \left( \vec{D}_{(1)} \right) = \rho_{(1)} \]
Order one charge and current equilibrium

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Order one charge and current equilibrium

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- \( \sigma I \left( N_D - N_D^+ \right) = \xi n_\text{e} N_D^+ \)
- \( \sigma I_1 \left( N_D - N_D^+(0) \right) + \sigma I_0 \left( -N_D^+(1) \right) = \)
  \[ \xi n_\text{e}(0) N_D^+(1) + \xi n_\text{e}(1) N_D^+(0) \]

Harmonic Current density

\( \vec{J}_1 \) = \( \mu_\text{e} k_B T \text{grad} \left( n_\text{e} \right)_1 + e \mu_\text{e} n_\text{e}(1) \vec{E}_1 \)

Harmonic Current density divergence is null

\( i \vec{K} \cdot \vec{J} = 0 \)

\( i \vec{K} \cdot \left( \mu_\text{e} k_B T n_\text{e}(1) \vec{K} + e \mu_\text{e} n_\text{e}(1) \vec{E}_1 \right) = 0 \)

Harmonic Poisson

\( \text{div} \left( \vec{D}_1 \right) = \rho_1 \)
Order one charge and current equilibrium

Steady state equilibrium

- \( \sigma I \left( N_D - N_D^+ \right) = \xi n_\text{e} N_D^+ \)
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Harmonic Current density

\[ \vec{J}_\text{(1)} = \mu_\text{e} k_B T i n_\text{e}(1) \vec{K} + e \mu_\text{e} n_\text{e}(1) \vec{E}_\text{(1)} \]

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Order one charge and current equilibrium

Steady state equilibrium

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- \( \sigma I_{(1)} (N_D - N_D^{(0)}) + \sigma I_{(0)} (-N_D^{(1)}) = \)
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- $\sigma I_{(1)} \left( N_D - N_D^{+\, (0)} \right) + \sigma I_{(0)} \left( -N_D^{+\, (1)} \right) = \xi n_\varepsilon^{(0)} N_D^{+\, (1)} + \xi n_\varepsilon^{(1)} N_D^{+\, (0)}$

Harmonic Current density

$$\vec{J}_{(1)} = \mu_\varepsilon k_B T i n_\varepsilon^{(1)} \vec{K} + e\mu_\varepsilon n_\varepsilon^{(1)} \vec{E}_{(1)}$$

Harmonic Current density divergence is null

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Harmonic Poisson

$$\text{div} \left( \vec{D}_{(1)} \right) = \rho_{(1)}$$
### Order one charge and current equilibrium

#### Steady state equilibrium

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#### Harmonic Current density

$$ \overrightarrow{J}_{(1)} = \mu_\varepsilon k_B T \dot{n}_\varepsilon{(1)} \overrightarrow{K} + e \mu_\varepsilon n_\varepsilon{(1)} \overrightarrow{E}_{(1)} $$

#### Harmonic Current density divergence is null

$$ i \overrightarrow{K} \cdot \overrightarrow{J} = 0 $$

$$ i \overrightarrow{K} \cdot \left( \mu_\varepsilon k_B T \dot{n}_\varepsilon{(1)} \overrightarrow{K} + e \mu_\varepsilon n_\varepsilon{(1)} \overrightarrow{E}_{(1)} \right) = 0 $$

#### Harmonic Poisson

$$ i \overrightarrow{K} \cdot \left( \hat{\varepsilon} \cdot \overrightarrow{E}_{(1)} \right) = e \left( N_D^{(1)} - n_\varepsilon{(1)} \right) $$
Order One Space Charge Field

**General Expression**

\[
\vec{E}^{(1)} = \frac{\hat{K} \frac{k_B T}{e} - \hat{K} \cdot \mu E^{(0)} \langle \mu \rangle}{1 + \frac{\|\hat{K}\|^2}{k_D^2} + i \frac{E^{(0)} \cdot \hat{K} \cdot \mu \langle \mu \rangle}{k_D^2 \langle \mu \rangle}} \frac{I^{(1)}}{I^{(0)}}
\]

**Effective permittivity**

\[
\langle \varepsilon \rangle = \frac{\hat{K} \cdot \varepsilon \hat{K}}{\|\hat{K}\|^2}
\]

**Effective permeability**

\[
\langle \mu \rangle = \frac{\hat{K} \cdot \mu \hat{K}}{\|\hat{K}\|^2}
\]

**Debye vector**

\[
k_D^2 = \frac{e}{\langle \varepsilon \rangle} \frac{e}{k_B T} \frac{N_D}{N_A} (N_D - N_A)
\]

\[
k_D = \frac{2\pi}{\lambda_D}
\]
1. Band Transport Model
   - Schematics
   - Carrier Generation
   - Charge Transport
   - Electro-optic effect

2. Harmonic illumination
   - Harmonic framework
   - Uniform background: order 0
   - Periodic modulation: order 1
   - Implications, Simplifications, Diffusion and Saturation

3. Two Wave Mixing
   - Gratings graphical view
   - Coupled waves
   - Two Beam Coupling
Let’s simplify this complex expression

**General Expression**

\[
\vec{E}_1 = \frac{i \vec{K} \frac{k_B T}{e} - \frac{\vec{K} \cdot \mu E_0}{\vec{K} <\mu>} \frac{I_1}{I_0}}{1 + \frac{||\vec{K}||^2}{k_D^2} + i \frac{e}{k_B T} \frac{\vec{K} \cdot \mu E_0}{k_D^2 <\mu>} \frac{I_1}{I_0}}
\]

**Simplifying assumptions**

- Very often \( \vec{E}_1 \parallel \vec{K} \)
- When no field is applied : \( \vec{E}_0 = 0 \)
- Quarter period phase shift between Intensity and Space-Charge Field gratings
Let’s simplify this complex expression

**General Expression**

\[
\vec{E}(1) = \frac{i\vec{K} \frac{k_B T}{e} - \frac{\vec{K} \cdot E(0)}{k} I(1)}{1 + \left\| \vec{K} \right\|^2} + \frac{e k_B I(0)}{k^2} \frac{\vec{K} \cdot E(0)}{k^2} \frac{I(1)}{I(0)}
\]

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**General Expression**

\[
\vec{E}^{(1)} = \frac{i \vec{K} \frac{k_B T}{e} I^{(1)}}{1 + \frac{\|\vec{K}\|^2}{k_D^2} I^{(0)}}
\]

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Space charge field vs. grating spacing
\[ \Lambda = \frac{2\pi}{\| K \|} \]

**Large grating spacing**

- Small \( \vec{K} \)
  
  \[ \vec{E} (1) = i \vec{K} \frac{k_B T}{e} \frac{I(1)}{I(0)} \]

- Diffusion field: \( \vec{E}_d = \vec{K} \frac{k_B T}{e} \)

**Small grating spacing**

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  \[ \vec{E} (1) = i \vec{K} \frac{k_B T}{e} \frac{k_D^2}{\| K \|^2} \frac{I(1)}{I(0)} \]

- Saturation Field: \( \vec{E}_q = \vec{K} \frac{k_B T}{e} \frac{k_D^2}{\| K \|^2} \)
Space charge field vs. grating spacing

\[ \Lambda = 2\pi / \| \mathbf{K} \| \]

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- Saturation Field: \( \mathbf{E}_q = \mathbf{K} \frac{k_B T}{e} \frac{k_D^2}{\| \mathbf{K} \|^2} \)
Space charge field as a function of grating spacing
Space charge field with externally applied field

No applied field

\[ \vec{E}_{(1)} = i \frac{\vec{E}_d}{1 + \frac{E_d}{E_q}} \frac{\mathcal{I}_{(1)}}{\mathcal{I}_{(0)}} \]

Applied field \( \vec{E}_a \)

\[ \vec{E}_{(1)} = i \frac{\vec{E}_d}{1 + \frac{E_d}{E_q}} \left[ 1 + i \frac{E_a}{E_d + E_q} \right] \frac{\mathcal{I}_{(1)}}{\mathcal{I}_{(0)}} \]
Space charge field with externally applied field

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Applied field effect

Standard approximations

- For most gratings and materials: $E_d \ll E_q$
- Applied field in the middle: $E_d \ll E_a \ll E_q$

In phase\(^1\) illumination and space charge gratings

$$\vec{E}^{(1)} = i \frac{\vec{E}_d}{1 + \frac{E_d}{E_q} \left[ i \frac{E_a}{E_d} \right]} \frac{\mathcal{I}^{(1)}}{\mathcal{I}^{(0)}}$$

\(^1\)Actually, they are $\pi$ phase shifted. A possible negative sign on the electro-optic coefficient renders in-phase index and illumination gratings.
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$$\vec{E}(1) = i \frac{\vec{E}_d}{1 + \frac{E_d}{E_q}} \left[ i \frac{E_a}{E_d} \frac{\mathcal{I}(1)}{\mathcal{I}(0)} \right]$$

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In phase\(^1\) illumination and space charge gratings

\[
\overrightarrow{E}_1^{(1)} = - \frac{\overrightarrow{E}_a}{1 + \frac{E_d}{E_q} \mathcal{I}(0)} \mathcal{I}(1)
\]

---

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In phase intensity and index gratings
Beam interference is destructive, owing to reflection sign reversal
Quarter period shifted gratings
Beam interference is constructive
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Two waves and a grating

Two waves make an intensity grating

- Waves are coherent and same wavelength
- Wave vectors are $\vec{k}_1$ and $\vec{k}_2$
- Intensity grating vector is $\vec{K} = \vec{k}_2 - \vec{k}_1$
- Waves amplitudes are $A_i = \sqrt{I_i} e^{-i\psi_i}$

Index Grating

- Assume $E_d \ll E_a \ll E_q$
- Index grating $\propto \Phi$ shifted illumination grating

$$n = n(0) + R e \left[ n(1) e^{i\Phi} A_1 A_2 \frac{e^{\vec{K} \cdot \vec{r}}}{I(0)} \right]$$

- $\Phi = \pi/2$ if no applied field and $\Phi = 0$ if field applied
Two waves and a grating

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$$n = n(0) + \text{Re} \left[ n(1)e^{i\Phi}\frac{A_1A_2}{I(0)}e^{\vec{K}\cdot \vec{r}} \right]$$

- $\Phi = \pi/2$ if no applied field and $\Phi = 0$ if field applied
Assumption framework

- Propagation equation: \( \Delta A + \frac{\omega^2}{c^2} n^2 A = 0 \)
- SVA: \( \| \frac{\partial^2 A}{\partial z^2} \| \ll \| \beta \frac{\partial A}{\partial z} \| \)
- \( \beta \) such as \( \beta z = \vec{k} \cdot \vec{r} \)
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Co-propagative coupling $\beta_1 \beta_2 > 0$

Conventions

- $z = 0$: entrance in the photorefractive material
- Symmetric coupling: $\beta_1 = \beta_2 = \| \vec{k} \| \cos(\theta)$
  - $\theta$ is the half angle between input beams

After Coupled Mode calculations\(^2\)

- $\frac{\partial A_1}{\partial z} = -\frac{1}{2I(0)} \Gamma \| A_2 \|^2 A_1 - \alpha A_1$
- $\frac{\partial A_2}{\partial z} = -\frac{1}{2I(0)} \Gamma \| A_1 \|^2 A_2 - \alpha A_2$
- $\Gamma = i \frac{2\pi n(1)}{\lambda \cos(\theta)} e^{-i\Phi}$
- $\alpha$ is absorption

---
\(^2\)See lessons on Second Harmonic Generation and Optical Phase Conjugation for details.
Co-propagative coupling \( \beta_1 \beta_2 > 0 \)

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**Intensity and phase coupling**

Diffusion induces intensity coupling

Drift induces phase coupling

---

**Separate Diffusion and Drift influences**

\[ \Gamma = \gamma + 2i\zeta \]

\[ \gamma = \frac{2\pi n(1)}{\lambda \cos(\theta)} \sin (\Phi) \]

\[ \zeta = \frac{\pi n(1)}{\lambda \cos(\theta)} \cos (\Phi) \]

---

### Intensity coupling

\[
\begin{align*}
\frac{\partial I_1}{\partial z} &= -\gamma \frac{I_1 I_2}{I_1 + I_2} - \alpha I_1 \\
\frac{\partial I_2}{\partial z} &= +\gamma \frac{I_1 I_2}{I_1 + I_2} - \alpha I_2
\end{align*}
\]

### Phase coupling

\[
\begin{align*}
\frac{\partial \psi_1}{\partial z} &= \zeta \frac{I_2}{I_1 + I_2} \\
\frac{\partial \psi_2}{\partial z} &= \zeta \frac{I_1}{I_1 + I_2}
\end{align*}
\]

---

### Energy transfer

- For small absorption \( \alpha \), energy is transferred from one beam to the other.
- Transfer direction is given by sign of \( \gamma \).
Intensity and phase coupling
Diffusion induces intensity coupling
Drift induces phase coupling

Separate Diffusion and Drift influences
\[ \Gamma = \gamma + 2i\zeta \]
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\[ \frac{\partial \psi_1}{\partial z} = \zeta \frac{I_2}{I_1 + I_2} \]

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Intensity and phase coupling
Diffusion induces intensity coupling

γ = \frac{2\pi n^{(1)}}{\lambda \cos(\theta)} \sin(\Phi)
ζ = \frac{\pi n^{(1)}}{\lambda \cos(\theta)} \cos(\Phi)

\Gamma = \gamma + 2i\zeta

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\frac{\partial I_1}{\partial z} = -\gamma \frac{I_1 I_2}{I_1 + I_2} - \alpha I_1
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\frac{\partial \psi_1}{\partial z} = \zeta \frac{I_2}{I_1 + I_2}
\frac{\partial \psi_2}{\partial z} = \zeta \frac{I_1}{I_1 + I_2}

Energy transfer
- For small absorption \( \alpha \), energy is transferred from one beam to the other
- Transfer direction is given by sign of \( \gamma \)
Photorefractive Two Wave Mixing

Coupled Modes Solution

- Let \( m = \frac{\mathcal{I}_1 (0)}{\mathcal{I}_2 (0)} \)

- \( \mathcal{I}_1 (z) = \mathcal{I}_1 (0) \frac{1 + m^{-1}}{1 + m^{-1} e^{\gamma z}} e^{-\alpha z} \)

- \( \mathcal{I}_2 (z) = \mathcal{I}_2 (0) \frac{1 + m}{1 + m e^{-\gamma z}} e^{-\alpha z} \)
Two Wave Mixing Intensity Coupling
Two Wave Mixing Intensity Coupling with Absorption