Photorefractivity

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Photorefractivity

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Photorefractive effect

History began in 1966 as *Optical Damage*

Optical Damage in $\mathrm{LiNbO_3}$

- Shine a light on $\mathrm{LiNbO_3}$
- Remove it
- Shine another : damaged crystal

Semi-permanent effect

- Leave it in the dark : still damaged
- Leave it under uniform light : sometimes repaired

Today

- Photorefractivity can prove useful
- Some people still call it *optical damage*:
  - Bad for linear optics (electro-optic modulators. . .)
  - Bad for *instant* Non Linear Optics (SHG,OPA. . .)

Photorefractivity is attractive

Non linear optics at low optical power

Non Linear Optics

- Dynamic Holography
- Phase conjugation
- All optical computing
- ...
- At milliwatts and below power levels

Observed in many non linear crystals

- Sillenites : $Bi_{12}SiO_{20}$, $Bi_{12}TiO_{20}$, $Bi_{12}GeO_{20}$
- Tungsten-Bronze : $Sr_xBa_{1-x}Nb_2O_6$
- Ferroelectrics : $LiNbO_3$, $BaTiO_3$
- Semiconductors : InP:Fe, AsGa

Usefull reading...
[Yeh93, GH88, GH89]

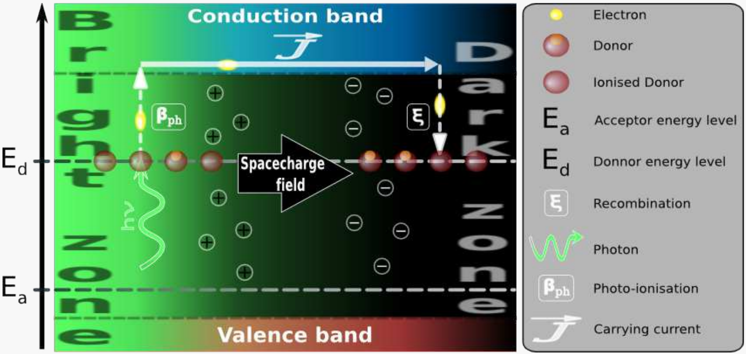
P. Günter and J. P. Huignard.
Photorefractive materials and their applications I, volume 61 of *Topics in Applied Physics*.
Springer Verlag, Berlin, 1988.

P. Günter and J. P. Huignard.
Photorefractive materials and their applications II, volume 62 of *Topics in Applied Physics*.
Springer Verlag, Berlin, 1989.

P. Yeh.
Introduction to photorefractive nonlinear optics.
Wiley Interscience, New York, 1993.

Contents

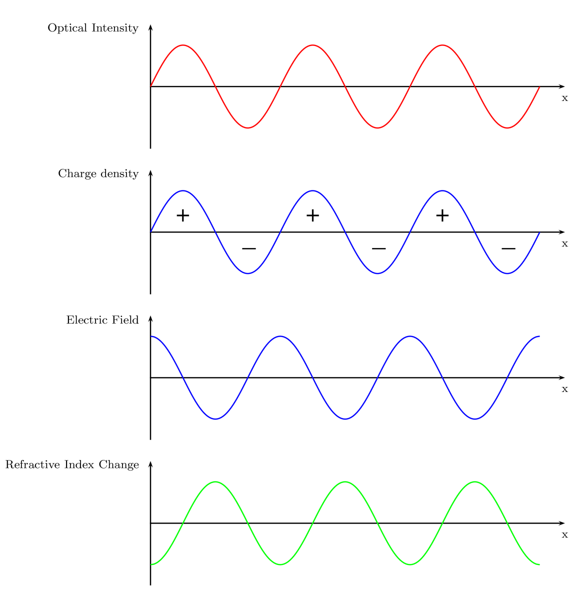
Photorefractive charge transport and trapping



Linear Index Modulation

Space charge electric field generates refractive index variation through electro-optic effect

Periodic illumination of a photorefractive material



Donors wanted

Carriers are generated by donors: no donors, no carriers

Nominally pure crystals

- No in-band-gap level
  - No donor nor acceptor
  - No photorefractive effect
- Structural defects often present
- As well as pollutants
- They create in-band-gap levels
- Photorefractivity can arise from them

More efficient: doping

- Introduce in-band-gap species
- $LiNbO_3$:Fe, InP:Fe...

Introducing Donors and Acceptors

Introduce Donors of electrons N_D

- Energy level close to conduction band
- They easily give electrons to conduction band

Introducing Acceptors $N_A \ll N_D$

- Photorefractivity needs traps
- Ionized donors are traps
- Introduce Acceptors close to the valence band
- They catch Donors electrons
- Donors are partially ionized $N_D^+ = N_A$

Thermal carrier generation

We assume here that the only carriers are electrons. . . what if not?

$n_{\mathring{e}}$

Comes from temperature induced Brownian motion

- Temperature induced
- Electrons are kicked into conduction band

Rate proportional to donors-left-to-ionize density

$$\frac{\partial n_{\mathring{e}}}{\partial t} = \beta \left(N_D - N_D^+ \right)$$

One generated electron leaves one ionized donor

$$\frac{\partial n_{\mathring{e}}}{\partial t} = \frac{\partial N_D^+}{\partial t} = \beta \left(N_D - N_D^+ \right)$$

Photo-excitation of carriers

The photoelectric effect at work

Photoelectric effect

- Photon energy sufficient to reach conduction band
- Rate proportional to light intensity $\mathcal{I}$
- And to left-to-ionize donors

Photo-excitation rate

$$\frac{\partial n_{\mathring{e}}}{\partial t} = \frac{\partial N_D^+}{\partial t} = \sigma \mathcal{I} \left(N_D - N_D^+ \right)$$

The photo-ionization cross section σ

- Has the dimensions of a surface
- If $\mathcal{I}$ is given as a number of photon per surface units and time
- Sometimes the case, sometimes not...

Carrier recombination

Recombination needs luck, electrons and empty traps

- A luck factor ξ
- Carriers density $n_{\mathring{e}}$
- Empty trap density N_D^+

$$\frac{\partial n_{\mathring{e}}}{\partial t} = \frac{\partial N_D^+}{\partial t} = -\xi n_{\mathring{e}} N_D^+$$

Carriers rate equation
A combination of generation and recombination

A combination of generation and recombination

$$\frac{\partial N_D^+}{\partial t} = (\beta + \sigma \mathcal{I}) \left(N_D - N_D^+ \right) - \xi n_{\mathring{e}} N_D^+$$

Charge transport in the conduction band

An assumption of the Band Transport Model…sometimes untrue

Diffusion

- Due to Temperature and Brownian motion
- Think of it as sugar in water (or coffee)
- Depends on concentration variations

Drift under electric-field

- Needs electric-field
- Externally applied or diffusion generated
- Depends on electric field and mobility

Photovoltaic effect

- Sometimes called *photo-galvanic*
- Non-isotropic effect
- Think of solar cells: light generates current
- Depends on light intensity

Diffusion transport

Diffusion current from Fick's first law linked to Einstein relation

Fick's first law — Particle flow

$$\overrightarrow{J_p} = -D\,\mathrm{grad}\,(p)$$

Einstein relation

- Links diffusion, absolute temperature T and Brownian motion through mobility
- Mobility μ is the velocity to electric field ratio

$$D = \mu_{\mathring{e}} k_B T / e$$

Diffusion Current

$$\overrightarrow{\mathcal{J}} = -e\overrightarrow{J_{\mathring{e}}} = +\mu_{\mathring{e}} k_B T\,\mathrm{grad}\,(n_{\mathring{e}})$$

Drift

A charged particle in an electric field. . .

Electric Field $\overrightarrow{E}$

- Externally applied
- Due to charged carrier diffusion

Drift current

- Electrons velocity: $\overrightarrow{v} = -\mu_{\mathring{e}} \overrightarrow{E}$
- Drift Current: $\overrightarrow{\mathcal{J}} = -e \times \overrightarrow{v}$

$$\overrightarrow{\mathcal{J}} = e n_{\mathring{e}} \mu_{\mathring{e}} \overrightarrow{E}$$

Photovoltaic current

An non isotropic effect stemming from crystal asymmetry

Origins

- Non centro-symmetric crystal *e.g.* $LiNbO_3$
- Anisotropic photo-electric effect
- Depends on light polarization

Photovoltaic tensor

- Rank 2
- Main component along polar axis
- Often reduced to a scalar

Photovoltaic Current

$$\left[\vec{\mathcal{J}}\right]_i = \left(N_D - N_D^+\right) \sum_{j,k} \left[\beta^{ph}\right]_{j,k} \left[\vec{E}\right]_j \left[\vec{E}\right]_k \vec{u_i}$$

$$\vec{\mathcal{J}} \approx \beta^{ph} \mathcal{I} \left(N_D - N_D^+\right) \vec{c}$$

Band Transport Model

Also known as Kukhtarev model

Published in 1979

Ionized donors rate equation

$$\frac{\partial N_D^+}{\partial t} = (\beta + \sigma \mathcal{I}) \left(N_D - N_D^+\right) - \xi n_{\mathring{e}} N_D^+$$

Current density expression

$$\overrightarrow{\mathcal{J}} = \mu_{\mathring{e}} k_B T \operatorname{grad}\left(n_{\mathring{e}}\right) + e n_{\mathring{e}} \mu_{\mathring{e}} \overrightarrow{E} + \beta^{ph} \mathcal{I} \left(N_D - N_D^+\right) \overrightarrow{c}$$

Quasi-static Maxwell model

- Continuity : $\operatorname{div}\left(\overrightarrow{\mathcal{J}}\right) + \frac{\partial \rho}{\partial t} = 0$
- Charge : $\rho = e\left(N_D^+ - N_A^- - n_{\mathring{e}}\right)$
- Maxwell-Gauss : $\operatorname{div}\left(\overrightarrow{D}\right) = \rho$, with $\overrightarrow{D} = \widehat{\varepsilon}\overrightarrow{E}$

Refractive index modulation through electro-optics

Space-charge electric field induces refractive index variations

This is not an electro-optics lesson

Please refer to the electro-optics lesson

Anyhow. . .

- Light generated electric field: the space charge field $\overrightarrow{E}$
- In electro-optic materials : creates index modulation

$$\left[\Delta \frac{1}{n^2}\right]_{i,j} = \sum_k [\widehat{r}]_{ijk} \left[\overrightarrow{E}\right]_k$$

- Local modulation of refractive index
- Local modification of refractive index ellipsoïd

Periodic illumination from plane waves interference

Two plane waves interfering

- Same wavelength and coherent
- non collinear wave vectors $\overrightarrow{k_1}$ and $\overrightarrow{k_2}$: $\overrightarrow{K} = \overrightarrow{k_2} - \overrightarrow{k_1}$
- Interference pattern : $\mathcal{I}_{(0)} \left(1 + m \cos\left(\overrightarrow{K} \cdot \overrightarrow{r}\right)\right)$
  - $\mathcal{I}_{(0)} = \mathcal{I}_1 + \mathcal{I}_2$
  - $m = 2\frac{\sqrt{\mathcal{I}_1 \mathcal{I}_2}}{\mathcal{I}_1 + \mathcal{I}_2}$

Harmonic assumptions

- $m \ll 1$: intensities are very different
- All unknowns are sum of
  - A large uniform background : order 0
  - A small harmonic modulation : order 1
- Linearity : orders can be uncoupled
  - Uniform intensity analysis
  - Followed by small signal analysis

Simplifying assumptions

One dimension problem 1D

- Plane waves interference
- All phenomena are collinear to $\overrightarrow{K}$

Drift-diffusion transport only assumed

Photovoltaic effect assumed negligible

Photo-generation only assumed

Large intensities: thermal generation can be neglected

Steady state study

All temporal derivatives assumed zero

Carrier generation–recombination equilibrium

Steady state equilibrium

$$\sigma\mathcal{I}\left(N_D - N_D^+\right) = \xi n_{\mathring{e}} N_D^+$$

Uniform electric field

- $\overrightarrow{D}_{(0)}$ is homogeneous
- $\mathrm{div}\left(\overrightarrow{D}_{(0)}\right) = \rho_{(0)} = 0$
- $N^+_{D\,(0)} - N_A^- - n_{\mathring{e}(0)} = 0$

Small illumination

- $n_{\mathring{e}} \ll N_A$
- $\sigma\mathcal{I} \ll \xi N_A$

Equilibrium homogeneous densities

- $N^+_{D\,(0)} = N_A + n_{\mathring{e}(0)}$
- $n_{\mathring{e}(0)} = \frac{N_D - N_A}{\xi N_A}\sigma\mathcal{I}_{(0)}$

Order 1 framework

Basic multi-scale modeling

All quantities are assumed periodic

- $\mathrm{div}\left(\overrightarrow{X}\right) = i\overrightarrow{K} \cdot \overrightarrow{X}$
- ...

Order 0 assumed known — Order 1 assumed small

- $\forall X,\ X_{(1)} \ll X_{(0)}$
- $(X \times Y)_{(1)} = X_{(0)} Y_{(1)} + X_{(1)} Y_{(0)}$
- Order 0 is independently found
- Order 1 products is Order 2 : assumed negligible

Order one charge and current equilibrium

Steady state equilibrium

- $\sigma\mathcal{I}\left(N_D - N_D^+\right) = \xi n_{\mathring{e}} N_D^+$
- $\sigma\mathcal{I}_{(1)}\left(N_D - N^+_{D\,(0)}\right) + \sigma\mathcal{I}_{(0)}\left(-N^+_{D\,(1)}\right) = \xi n_{\mathring{e}(0)} N^+_{D\,(1)} + \xi n_{\mathring{e}(1)} N^+_{D\,(0)}$

Harmonic Current density

$$\overrightarrow{\mathcal{J}}_{(1)} = \mu_{\mathring{e}} k_B T i n_{\mathring{e}(1)} \overrightarrow{K} + e\mu_{\mathring{e}} n_{\mathring{e}(1)} \overrightarrow{E}_{(1)}$$

Harmonic Current density divergence is null $\quad i\overrightarrow{K}\cdot\overrightarrow{\mathcal{J}} = 0$

$$i\overrightarrow{K}\cdot\left(\mu_{\mathring{e}} k_B T i n_{\mathring{e}(1)} \overrightarrow{K} + e\mu_{\mathring{e}} n_{\mathring{e}(1)} \overrightarrow{E}_{(1)}\right) = 0$$

Harmonic Poisson

$$i\overrightarrow{K}\cdot\left(\widehat{\varepsilon}\cdot\overrightarrow{E}_{(1)}\right) = e\left(N^+_{D\,(1)} - n_{\mathring{e}(1)}\right)$$

Order One Space Charge Field

General Expression

$$\overrightarrow{E}_{(1)} = \frac{i\overrightarrow{K}\frac{k_B T}{e} - \frac{\overrightarrow{K}\cdot\mu E_{(0)}}{\overrightarrow{K}<\mu>}}{1 + \frac{\|\overrightarrow{K}\|^2}{k_D^2} + i\frac{e}{k_B T}\frac{\overrightarrow{K}\cdot\mu E_{(0)}}{k_D^2<\mu>}}\frac{\mathcal{I}_{(1)}}{\mathcal{I}_{(0)}}$$

Effective permittivity

$$<\varepsilon>= \frac{\overrightarrow{K}\cdot\widehat{\varepsilon}\overrightarrow{K}}{\|\overrightarrow{K}\|^2}$$

Effective permeability

$$<\mu>= \frac{\overrightarrow{K}\cdot\mu\overrightarrow{K}}{\|\overrightarrow{K}\|^2}$$

Debye vector $k_D = \frac{2\pi}{\lambda_D}$

$$k_D^2 = \frac{e}{<\varepsilon>}\frac{e}{k_B T}\frac{N_D}{N_A}(N_D - N_A)$$

Let's simplify this complex expression

General Expression

$$\overrightarrow{E}_{(1)} = \frac{i\overrightarrow{K}\,\frac{k_B T}{e}}{1 + \frac{\|\overrightarrow{K}\|^2}{k_D^2}} \frac{\mathcal{I}_{(1)}}{\mathcal{I}_{(0)}}$$

Simplifying assumptions

- Very often $\overrightarrow{E}_{(1)} \parallel \overrightarrow{K}$
- When no field is applied : $\overrightarrow{E}_{(0)} = 0$
- Quarter period phase shift between Intensity and Space-Charge Field gratings

Space charge field *vs.* grating spacing $\Lambda = 2\pi / \| \overrightarrow{K} \|$

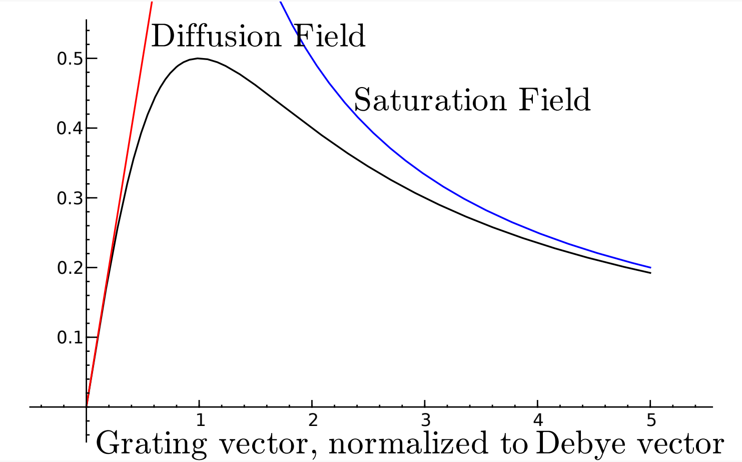
Large grating spacing

- Small $\overrightarrow{K}$
- $\overrightarrow{E}_{(1)} = i\overrightarrow{K}\frac{k_B T}{e}\frac{\mathcal{I}_{(1)}}{\mathcal{I}_{(0)}}$
- Diffusion field : $\overrightarrow{E_d} = \overrightarrow{K}\frac{k_B T}{e}$

Small grating spacing

- Large $\overrightarrow{K}$
- $\overrightarrow{E}_{(1)} = i\overrightarrow{K}\frac{k_B T}{e}\frac{k_D^2}{\| \overrightarrow{K} \|^2}\frac{\mathcal{I}_{(1)}}{\mathcal{I}_{(0)}}$
- Saturation Field : $\overrightarrow{E_q} = \overrightarrow{K}\frac{k_B T}{e}\frac{k_D^2}{\| \overrightarrow{K} \|^2}$

Space charge field as a function of grating spacing



Space charge field with externally applied field

No applied field

$$\overrightarrow{E}_{(1)} = i \frac{\overrightarrow{E_d}}{1 + \frac{E_d}{E_q}} \frac{\mathcal{I}_{(1)}}{\mathcal{I}_{(0)}}$$

Applied field $\overrightarrow{E_a}$

$$\overrightarrow{E}_{(1)} = i \frac{\overrightarrow{E_d}}{1 + \frac{E_d}{E_q}} \left[\frac{1 + i \frac{E_a}{E_d}}{1 + i \frac{E_a}{E_d + E_q}} \right] \frac{\mathcal{I}_{(1)}}{\mathcal{I}_{(0)}}$$

Applied field effect

Standard approximations

- For most gratings and materials : $E_d \ll E_q$
- Applied field in the middle : $E_d \ll E_a \ll E_q$

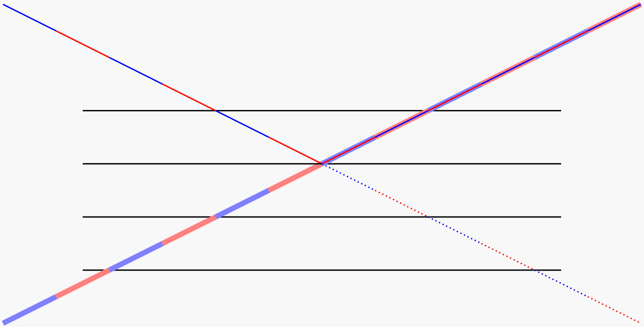
In phase[1] illumination and space charge gratings

$$\overrightarrow{E}_{(1)} = -\frac{\overrightarrow{E_a}}{1+\frac{E_d}{E_q}} \frac{\mathcal{I}_{(1)}}{\mathcal{I}_{(0)}}$$

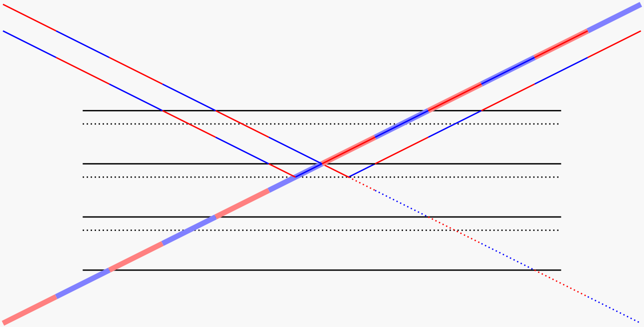
[1] Actually, they are π phase shifted. A possible negative sign on the electro-optic coefficient renders in-phase index and illumination gratings.

In phase intensity and index gratings

Beam interference is destructive, owing to reflection sign reversal



Quarter period shifted gratings
Beam interference is constructive



Two waves and a grating

Two waves make an intensity grating

- Waves are coherent and same wavelength
- Wave vectors are $\overrightarrow{k_1}$ and $\overrightarrow{k_2}$
- Intensity grating vector is $\overrightarrow{K} = \overrightarrow{k_2} - \overrightarrow{k_1}$
- Waves amplitudes are $A_i = \sqrt{\mathcal{I}_i} e^{-i\psi_i}$

Index Grating

- Assume $E_d \ll E_a \ll E_q$
- Index grating $\propto$ Φ shifted illumination grating

$$n = n_{(0)} + \mathcal{R}e\left[n_{(1)} e^{i\Phi} \frac{\overline{A_1} A_2}{\mathcal{I}_{(0)}} e^{\overrightarrow{K} \cdot \overrightarrow{r}}\right]$$

- $\Phi = \pi/2$ if no applied field and $\Phi = 0$ if field applied

Assumption framework

Slow Varying Approximation **Paraxial Framework**

- Propagation equation : $\Delta A + \frac{\omega^2}{c^2} n^2 A = 0$
- SVA: $\|\frac{\partial^2 A}{\partial z^2}\| \ll \|\beta \frac{\partial A}{\partial z}\|$
- β such as $\beta z = \overrightarrow{k} \cdot \overrightarrow{r}$

Co-propagative coupling $\beta_1\beta_2 > 0$

Conventions

- $z = 0$: entrance in the photorefractive material
- Symmetric coupling : $\beta_1 = \beta_2 = \|\overrightarrow{k}\|\cos(\theta)$

  θ is the half angle between input beams

After Coupled Mode calculations[2]

- $\frac{\partial A_1}{\partial z} = -\frac{1}{2\mathcal{I}_{(0)}}\Gamma\|A_2\|^2 A_1 - \alpha A_1$
- $\frac{\partial A_2}{\partial z} = -\frac{1}{2\mathcal{I}_{(0)}}\overline{\Gamma}\|A_1\|^2 A_2 - \alpha A_2$
- $\Gamma = i\frac{2\pi n_{(1)}}{\lambda\cos(\theta)}e^{-i\Phi}$
- α is absorption

[2] See lessons on Second Harmonic Generation and Optical Phase Conjugation for details.

Separate Diffusion and Drift influences

$$\Gamma = \gamma + 2i\zeta$$

$$\gamma = \frac{2\pi n_{(1)}}{\lambda \cos(\theta)} \sin(\Phi) \qquad \zeta = \frac{\pi n_{(1)}}{\lambda \cos(\theta)} \cos(\Phi)$$

Intensity coupling

- $\frac{\partial \mathcal{I}_1}{\partial z} = -\gamma \frac{\mathcal{I}_1 \mathcal{I}_2}{\mathcal{I}_1 + \mathcal{I}_2} - \alpha \mathcal{I}_1$
- $\frac{\partial \mathcal{I}_2}{\partial z} = +\gamma \frac{\mathcal{I}_1 \mathcal{I}_2}{\mathcal{I}_1 + \mathcal{I}_2} - \alpha \mathcal{I}_2$

Phase coupling

- $\frac{\partial \psi_1}{\partial z} = \zeta \frac{\mathcal{I}_2}{\mathcal{I}_1 + \mathcal{I}_2}$
- $\frac{\partial \psi_2}{\partial z} = \zeta \frac{\mathcal{I}_1}{\mathcal{I}_1 + \mathcal{I}_2}$

Energy transfer

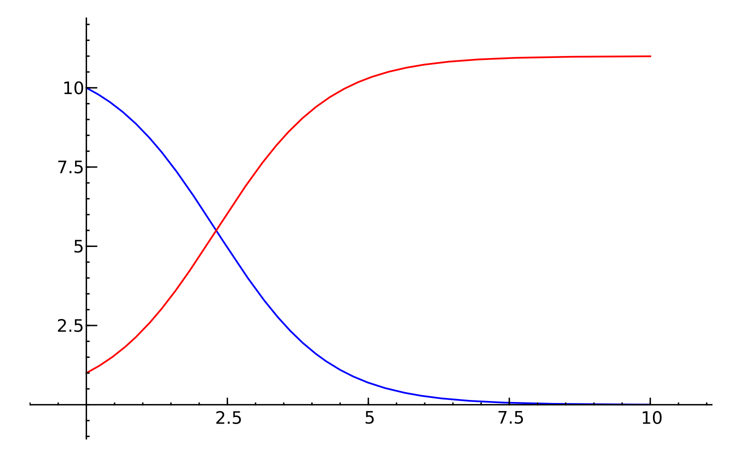
- For small absorption α, energy is transferred from one beam to the other
- Transfer direction is given by sign of γ

Photorefractive Two Wave Mixing

Coupled Modes Solution

- Let $m = \dfrac{\mathcal{I}_1(0)}{\mathcal{I}_2(0)}$
- $\mathcal{I}_1(z) = \mathcal{I}_1(0)\dfrac{1+m^{-1}}{1+m^{-1}e^{\gamma z}}e^{-\alpha z}$
- $\mathcal{I}_2(z) = \mathcal{I}_2(0)\dfrac{1+m}{1+me^{-\gamma z}}e^{-\alpha z}$

Two Wave Mixing Intensity Coupling



Two Wave Mixing Intensity Coupling with Absorption