The classification of subgroups of quantum SU(N)
Adrian Ocneanu

To cite this version:

HAL Id: cel-00374414
https://cel.archives-ouvertes.fr/cel-00374414
Submitted on 8 Apr 2009

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
The classification and structure of subgroups of quantum groups

Quantum Groups

The classification of subgroups of quantum groups

1. The classification and structure of subgroups of quantum groups
1.3. The classification and structure of quaternion subgroups of $\mathbb{S}(3)$.

Let $\mathbb{S}(4)$ and $\mathbb{S}(5)$ be the quaternion groups of order 16 and 32, respectively. The quaternion group $\mathbb{S}(4)$ is the group of eight elements, generated by two elements $i$ and $j$, with the relations $i^2 = j^2 = (ij)^2 = 1$. The quaternion group $\mathbb{S}(5)$ is the group of 16 elements, generated by two elements $i$ and $j$, with the relations $i^2 = j^2 = (ij)^2 = 1$.

The quaternion group $\mathbb{S}(4)$ is the group of eight elements, generated by two elements $i$ and $j$, with the relations $i^2 = j^2 = (ij)^2 = 1$. The quaternion group $\mathbb{S}(5)$ is the group of 16 elements, generated by two elements $i$ and $j$, with the relations $i^2 = j^2 = (ij)^2 = 1$.
The classification of structures of a given type is a fundamental problem in the study of graphs. Given any graph, the classification process involves identifying its structural properties and classifying it based on these properties. The problem is not trivial, especially when the graph's structure is complex.

Let's consider a graph $G = (V, E)$, where $V$ is the set of vertices and $E$ is the set of edges. The problem of classification involves determining the adjacency matrix $A$ of the graph, which is a square matrix where the entry $A_{ij}$ represents the number of edges between vertices $i$ and $j$.

For a graph with $n$ vertices, the adjacency matrix $A$ is an $n 	imes n$ matrix. The problem of classification is to find a method to determine the eigenvalues and eigenvectors of the adjacency matrix, which can provide insights into the structure of the graph.

The eigenvalues of the adjacency matrix $A$ are crucial in determining the graph's properties. They are the solutions to the characteristic equation $\det(A - \lambda I) = 0$, where $I$ is the identity matrix and $\lambda$ represents the eigenvalues.

Moreover, the eigenvectors corresponding to these eigenvalues provide additional information about the graph's structure. The eigenvectors can be used to classify the graph based on its connectivity, symmetry, and other properties.

In summary, the classification of graphs is a complex and important problem in graph theory. It involves understanding the graph's adjacency matrix, determining its eigenvalues and eigenvectors, and using these properties to classify the graph based on its structural characteristics.
The exception on page 8 of Theorem 4.3 is the exceptional subgroup $\mathcal{E}$. Thus, in the exceptional case, the rank of the group is $\text{rank}(\mathcal{E}) = 1$. In general, the rank of the group is $\text{rank}(\mathcal{E})$. The unique property of the group is that $\mathcal{E}$ is the non-exceptional case.

1. The non-exceptional case of $\mathcal{E}$.

2. The non-exceptional case of $\mathcal{E}$.

3. The non-exceptional case of $\mathcal{E}$.

4. The non-exceptional case of $\mathcal{E}$.

5. The non-exceptional case of $\mathcal{E}$.

6. The non-exceptional case of $\mathcal{E}$.

7. The non-exceptional case of $\mathcal{E}$.

8. The non-exceptional case of $\mathcal{E}$.

9. The non-exceptional case of $\mathcal{E}$.

10. The non-exceptional case of $\mathcal{E}$.

11. The non-exceptional case of $\mathcal{E}$.

12. The non-exceptional case of $\mathcal{E}$.

13. The non-exceptional case of $\mathcal{E}$.

14. The non-exceptional case of $\mathcal{E}$.

15. The non-exceptional case of $\mathcal{E}$.

16. The non-exceptional case of $\mathcal{E}$.

17. The non-exceptional case of $\mathcal{E}$.

18. The non-exceptional case of $\mathcal{E}$.

19. The non-exceptional case of $\mathcal{E}$.

20. The non-exceptional case of $\mathcal{E}$.

21. The non-exceptional case of $\mathcal{E}$.

22. The non-exceptional case of $\mathcal{E}$.

23. The non-exceptional case of $\mathcal{E}$.

24. The non-exceptional case of $\mathcal{E}$.

25. The non-exceptional case of $\mathcal{E}$.

26. The non-exceptional case of $\mathcal{E}$.

27. The non-exceptional case of $\mathcal{E}$.

28. The non-exceptional case of $\mathcal{E}$.

29. The non-exceptional case of $\mathcal{E}$.

30. The non-exceptional case of $\mathcal{E}$.

31. The non-exceptional case of $\mathcal{E}$.

32. The non-exceptional case of $\mathcal{E}$.

33. The non-exceptional case of $\mathcal{E}$.

34. The non-exceptional case of $\mathcal{E}$.

35. The non-exceptional case of $\mathcal{E}$.

36. The non-exceptional case of $\mathcal{E}$.

37. The non-exceptional case of $\mathcal{E}$.

38. The non-exceptional case of $\mathcal{E}$.

39. The non-exceptional case of $\mathcal{E}$.

40. The non-exceptional case of $\mathcal{E}$.

41. The non-exceptional case of $\mathcal{E}$.

42. The non-exceptional case of $\mathcal{E}$.

43. The non-exceptional case of $\mathcal{E}$.

44. The non-exceptional case of $\mathcal{E}$.

45. The non-exceptional case of $\mathcal{E}$.

46. The non-exceptional case of $\mathcal{E}$.

47. The non-exceptional case of $\mathcal{E}$.

48. The non-exceptional case of $\mathcal{E}$.

49. The non-exceptional case of $\mathcal{E}$.

50. The non-exceptional case of $\mathcal{E}$.
\[ M \geq \chi_{\nu}^2 \]
The problem, solved by Yutaka Jones, was to find a closed formula for the

expression of each monomial term in the expression of the commutator.

In general, a closed formula of the Weyl's operator commutes one

\[ \frac{1}{2} \frac{d}{dz} \mathcal{L} \left( \frac{d}{dz} \mathcal{L} \right) = \left( \frac{d}{dz} \mathcal{L} \right) \frac{d}{dz} \mathcal{L} \]

From this, we have

\[ \mathcal{L} \left( \frac{d}{dz} \mathcal{L} \right) = \left( \frac{d}{dz} \mathcal{L} \right) \frac{d}{dz} \mathcal{L} \]

Here, we prove the recurrence relation

\[ \mathcal{L} \left( \frac{d}{dz} \mathcal{L} \right) = \left( \frac{d}{dz} \mathcal{L} \right) \frac{d}{dz} \mathcal{L} \]

by induction on the index of the monomial. When the index is \( n \),

\[ \mathcal{L} \left( \frac{d}{dz} \mathcal{L} \right) = \left( \frac{d}{dz} \mathcal{L} \right) \frac{d}{dz} \mathcal{L} \]

The base case is trivial, and the inductive step follows by the recurrence relation

\[ \mathcal{L} \left( \frac{d}{dz} \mathcal{L} \right) = \left( \frac{d}{dz} \mathcal{L} \right) \frac{d}{dz} \mathcal{L} \]

The proof is complete.
Theorem A. The classification of groups of Gr"atam 

\[ (\mu_{\gamma}(1-\mu, \ldots, \mu)) \left( \prod_{\gamma=1}^{\infty} \right) \sum_{\gamma=1}^{\infty} (1-\mu) \delta_{1, \mu} = \delta_{d} \]
In a condensation, we have obtained a very fundamental and general construction of a functional basis in the sense of the $\mathcal{A}$-functions. This construction

In a condensation, we have obtained a very fundamental and general construction of a functional basis in the sense of the $\mathcal{A}$-functions. This construction

In a condensation, we have obtained a very fundamental and general construction of a functional basis in the sense of the $\mathcal{A}$-functions. This construction

In a condensation, we have obtained a very fundamental and general construction of a functional basis in the sense of the $\mathcal{A}$-functions. This construction

In a condensation, we have obtained a very fundamental and general construction of a functional basis in the sense of the $\mathcal{A}$-functions. This construction

In a condensation, we have obtained a very fundamental and general construction of a functional basis in the sense of the $\mathcal{A}$-functions. This construction

In a condensation, we have obtained a very fundamental and general construction of a functional basis in the sense of the $\mathcal{A}$-functions. This construction

In a condensation, we have obtained a very fundamental and general construction of a functional basis in the sense of the $\mathcal{A}$-functions. This construction

In a condensation, we have obtained a very fundamental and general construction of a functional basis in the sense of the $\mathcal{A}$-functions. This construction

In a condensation, we have obtained a very fundamental and general construction of a functional basis in the sense of the $\mathcal{A}$-functions. This construction

In a condensation, we have obtained a very fundamental and general construction of a functional basis in the sense of the $\mathcal{A}$-functions. This construction

In a condensation, we have obtained a very fundamental and general construction of a functional basis in the sense of the $\mathcal{A}$-functions. This construction
consisting of the "non-parallel" linear combinations of paths.

If we view the vertices of $G$ as irreducible representations of $S(2N)$, then the essential path subspace is the subspace of $N$-level tensor products with the irreducible $\sigma_i = e$.

Let $N_{\sigma_i} = \text{dim} E \text{Path}^{(n)}(\sigma_i)$. Then $N_{\sigma_i} \neq 0$ if and only if $\sigma_i$ is an irreducible representation of $S(2N)$.

The maximal length of essential paths corresponding to the fusion number $\sigma_i$ is thus the level $L = N - 2$, since $N_i = 0$ for $i > 2$. The essential paths are determined by the condition that they are free to each other, and in other words, if $P = P_{ij}$ is a column vector, then $P_{ij} = \delta_{ij}$.

$c_i^{(n)} = \delta_{n,i} \psi_{n}^{(i)}(\sigma_i^{(i)})$. Let $c_i^{(n)}$ denote the $i$-th component of $c_i^{(n)}$. Let $\beta = c_i^{(n)}$ denote the Hilbert-Schmidt inner product between the vertices $(\sigma_i, 0)$ and $(\sigma_j, 0)$.

The graph $G$ has now $|\mathcal{K}|$ vertices, corresponding to the vertices of $G$. We now define the essential path subspace $E \text{Path}^{(n)}(\sigma_i)$ as the subspace of $n$-level tensor products with the irreducible $\sigma_i = e$.

We shall in fact define the vertices as the $\mathcal{K}$ vertices. We now define the essential path subspace $E \text{Path}^{(n)}(\sigma_i)$ as the subspace of $n$-level tensor products with the irreducible $\sigma_i = e$.

The graph $G$ has now $|\mathcal{K}|$ vertices, corresponding to the vertices of $G$. We now define the essential path subspace $E \text{Path}^{(n)}(\sigma_i)$ as the subspace of $n$-level tensor products with the irreducible $\sigma_i = e$.
Theorem 1: The inner product of two vectors in a **m**-dimensional space is given by:

\[
(f, g) = \sum_{i=1}^{m} f_i g_i
\]

**Proof:** The inner product is defined as the sum of the products of corresponding components of the two vectors. Hence, the formula follows directly from the definition.

**Theorem 2:** The result of the inner product operation between two vectors in a **m**-dimensional space is a scalar.

**Proof:** The inner product of two vectors results in a single number, which is a scalar. This is consistent with the definition of the inner product as the sum of the products of corresponding components.

**Corollary:** The inner product is commutative, meaning that the order of the vectors does not affect the result.

**Proof:** By the properties of the inner product, it follows that \((f, g) = (g, f)\) for any vectors \(f, g\) in a **m**-dimensional space.

**Theorem 3:** The inner product is also linear in each argument, meaning that for any scalar \(c\) and vectors \(f, g\):

\[
(cf, g) = c(f, g)\]

**Proof:** This property follows from the definition of the inner product and the linearity of the sum.

**Theorem 4:** The inner product of a vector with itself is the square of the norm of the vector.

**Proof:** Using the definition of the inner product, we have:

\[
(f, f) = \sum_{i=1}^{m} f_i^2 = \|f\|^2
\]

This completes the proof of the theorems and their corollaries for the inner product in a **m**-dimensional space.
\[ q \in \left( b \right)^{\overrightarrow{\sigma}} \quad \text{where} \quad \sigma \in \mathcal{G} \]

The product is normalized (a number of \( (b)^{\overrightarrow{\sigma}} \) if \( \sigma \) is a polynomial in \( b \)). Then the product is

\[ (f \cdot g) \to \mathbb{F} \quad \text{where} \quad f \in \mathbb{F}, \quad g \in \mathbb{F} \]

The product is defined by \( f \cdot g = \mathbb{F} \quad \text{where} \quad f \in \mathbb{F}, \quad g \in \mathbb{F} \). The product is defined by \( f \cdot g = \mathbb{F} \quad \text{where} \quad f \in \mathbb{F}, \quad g \in \mathbb{F} \). The product is defined by \( f \cdot g = \mathbb{F} \quad \text{where} \quad f \in \mathbb{F}, \quad g \in \mathbb{F} \). The product is defined by \( f \cdot g = \mathbb{F} \quad \text{where} \quad f \in \mathbb{F}, \quad g \in \mathbb{F} \).

An element of the polynomial ring is a polynomial in \( \mathbb{F} \). A polynomial in \( \mathbb{F} \) is a polynomial in \( \mathbb{F} \). A polynomial in \( \mathbb{F} \) is a polynomial in \( \mathbb{F} \). A polynomial in \( \mathbb{F} \) is a polynomial in \( \mathbb{F} \).

\[ \dim H \quad \text{where} \quad \dim H \]

A polynomial in \( \mathbb{F} \) is a polynomial in \( \mathbb{F} \). A polynomial in \( \mathbb{F} \) is a polynomial in \( \mathbb{F} \). A polynomial in \( \mathbb{F} \) is a polynomial in \( \mathbb{F} \).

\[ \dim H \quad \text{where} \quad \dim H \]

A polynomial in \( \mathbb{F} \) is a polynomial in \( \mathbb{F} \). A polynomial in \( \mathbb{F} \) is a polynomial in \( \mathbb{F} \). A polynomial in \( \mathbb{F} \) is a polynomial in \( \mathbb{F} \).

\[ \dim H \quad \text{where} \quad \dim H \]

A polynomial in \( \mathbb{F} \) is a polynomial in \( \mathbb{F} \). A polynomial in \( \mathbb{F} \) is a polynomial in \( \mathbb{F} \). A polynomial in \( \mathbb{F} \) is a polynomial in \( \mathbb{F} \).

\[ \dim H \quad \text{where} \quad \dim H \]

A polynomial in \( \mathbb{F} \) is a polynomial in \( \mathbb{F} \). A polynomial in \( \mathbb{F} \) is a polynomial in \( \mathbb{F} \). A polynomial in \( \mathbb{F} \) is a polynomial in \( \mathbb{F} \).

\[ \dim H \quad \text{where} \quad \dim H \]

A polynomial in \( \mathbb{F} \) is a polynomial in \( \mathbb{F} \). A polynomial in \( \mathbb{F} \) is a polynomial in \( \mathbb{F} \). A polynomial in \( \mathbb{F} \) is a polynomial in \( \mathbb{F} \).

\[ \dim H \quad \text{where} \quad \dim H \]

A polynomial in \( \mathbb{F} \) is a polynomial in \( \mathbb{F} \). A polynomial in \( \mathbb{F} \) is a polynomial in \( \mathbb{F} \). A polynomial in \( \mathbb{F} \) is a polynomial in \( \mathbb{F} \).

\[ \dim H \quad \text{where} \quad \dim H \]

A polynomial in \( \mathbb{F} \) is a polynomial in \( \mathbb{F} \). A polynomial in \( \mathbb{F} \) is a polynomial in \( \mathbb{F} \). A polynomial in \( \mathbb{F} \) is a polynomial in \( \mathbb{F} \).

\[ \dim H \quad \text{where} \quad \dim H \]

A polynomial in \( \mathbb{F} \) is a polynomial in \( \mathbb{F} \). A polynomial in \( \mathbb{F} \) is a polynomial in \( \mathbb{F} \). A polynomial in \( \mathbb{F} \) is a polynomial in \( \mathbb{F} \).

\[ \dim H \quad \text{where} \quad \dim H \]

A polynomial in \( \mathbb{F} \) is a polynomial in \( \mathbb{F} \). A polynomial in \( \mathbb{F} \) is a polynomial in \( \mathbb{F} \). A polynomial in \( \mathbb{F} \) is a polynomial in \( \mathbb{F} \).

\[ \dim H \quad \text{where} \quad \dim H \]

A polynomial in \( \mathbb{F} \) is a polynomial in \( \mathbb{F} \). A polynomial in \( \mathbb{F} \) is a polynomial in \( \mathbb{F} \). A polynomial in \( \mathbb{F} \) is a polynomial in \( \mathbb{F} \).

\[ \dim H \quad \text{where} \quad \dim H \]
In order to make use of modes of \( \mathcal{O} \)-theory, which is a central problem in this field, we have developed a set of properties and techniques for the construction of GNPs. We have described a very simple and natural way the construction of GNPs is achieved.

\section{New Directions of Research}

The main question which follows is the existence of further modes of the interaction of right-handed charged particles with the \( \mathcal{O} \)-theory.

We have described a very simple and natural way the construction of GNPs is achieved.

\section{Conclusions}

The main question which follows is the existence of further modes of the interaction of right-handed charged particles with the \( \mathcal{O} \)-theory.

We have described a very simple and natural way the construction of GNPs is achieved.
In the field of quantum mechanics, the concept of quantum states and their evolution under the action of quantum operators is central. The Schrödinger equation plays a crucial role in describing these evolutions.

The solutions of the equation provide the probability amplitudes for the system, which can be used to calculate observable quantities. These solutions are often expressed in terms of wave functions, which are complex-valued functions.

The mathematical framework of quantum mechanics is based on Hilbert spaces, which are vector spaces equipped with an inner product. These spaces allow for the description of superpositions of states, which are fundamental to the interpretation of quantum mechanics.

In conclusion, the study of quantum mechanics is essential for understanding the fundamental behavior of matter and energy. It provides the theoretical foundation for modern physics and has applications in various fields, including materials science, chemistry, and information technology.
THE MODULAR HIERARCHY. Each module provides a modular hierarchy, which is
simply the collection of its own, its parent, and its children.

MODULAR HIERARCHY.

\[ \begin{align*}
\text{root module} & : \text{root} \\
\text{parent module} & : \text{parent} \\
\text{child module} & : \text{child}
\end{align*} \]

Each of these modules has its own hierarchy, and the hierarchy of the root module is the
complete modular hierarchy. The root module is the only module without a parent.

CONTRIBUTIONS. All exceptions are handled by the root module. Exceptions from any
other module are handled by the module that contains the root module.

EXAMPLES. For each module, there is an exception and a root module.

1. The root module of the root module is the root module itself.
2. The root module of a child module is the parent module.
3. The root module of a child of a child is the child module.
4. The root module of a child of a child of a child is that child module.

CHALLENGE. To create a modular hierarchy, you must create a graph of the hierarchy.

Each module in the hierarchy is represented by a node in the graph, and the
connection between modules is represented by edges. The root module is the
root node, and each child module is a child node.

The graph structure allows for easy traversal of the hierarchy, making it
possible to find the root module or any child module in a single step.

THE MODULAR HIERARCHY OF THE MODULAR HIERARCHY. Each module provides a modular hierarchy, which is
simply the collection of its own, its parent, and its children.

MODULAR HIERARCHY.

\[ \begin{align*}
\text{root module} & : \text{root} \\
\text{parent module} & : \text{parent} \\
\text{child module} & : \text{child}
\end{align*} \]

Each of these modules has its own hierarchy, and the hierarchy of the root module is the
complete modular hierarchy. The root module is the only module without a parent.

CONTRIBUTIONS. All exceptions are handled by the root module. Exceptions from any
other module are handled by the module that contains the root module.

EXAMPLES. For each module, there is an exception and a root module.

1. The root module of the root module is the root module itself.
2. The root module of a child module is the parent module.
3. The root module of a child of a child is the child module.
4. The root module of a child of a child of a child is that child module.

CHALLENGE. To create a modular hierarchy, you must create a graph of the hierarchy.

Each module in the hierarchy is represented by a node in the graph, and the
connection between modules is represented by edges. The root module is the
root node, and each child module is a child node.

The graph structure allows for easy traversal of the hierarchy, making it
possible to find the root module or any child module in a single step.
References

not all exceptional graphs come from configurations
modular invariants, and the exceptional $I_{x}$ of the $\mathbb{F}_{p}$
declassification showed that different graphs can share the same
3-dimensional $N_{L}$'s. The $\mathbb{F}_{p}$ declassification showed that different graphs can share the same
4-dimensional $N_{L}$'s. The author of the article equivalence theory for 2-dimensional
natural equivalence of the graphs is part of our maximal algebra theory, which is
connection between graphs and modular invariants was an open problem. The
a subset of the be proposed empirically by de Francesco and Zuber, the precise
are $S_{x}(\mathbb{F}_{p})$ and $C_{x}(\mathbb{F}_{p})$ for $S_{x}$/$C_{x}$ in our construction of small index subgraphs, the
the best that the consideration $N_{L}$ and $C_{x}$ are defined on the locus. The
the classification of subgroups of $\Gamma(\mathbb{F}_{p})$.
**SU(2)\textsubscript{k}\**

**Orbifold series**

\[ \begin{array}{cccccccc}
A_2 & A_3 & A_4 & A_5 & D_4 & D_5 & D_6 & D_7 \\
(A_1)(A_2) & (A_3) & (A_4) & (A_4/2) & (A_6/2) & (A_8/2) & (A_{10}/2) & \ldots
\end{array} \]

**Exceptionals**

\[ \begin{array}{cccc}
E_6 & E_7 & E_8 \\
(E_{10}) & ((A_{16}/2)^t) & (E_{28})
\end{array} \]

**Figure 1.** Classification of modules and subgroups of quantum SU(2).
\textbf{SU}(3)_k

\textbf{Orbifold series}

\begin{align*}
\begin{array}{cccccccc}
A_1 & A_2 & A_3 & A_4 & A_5 & A_6 & \ldots \\
\end{array}
\end{align*}

\textbf{Conjugate orbifold series}

\begin{align*}
\begin{array}{cccccccc}
A_1 & A_2 & A_3 & A_4 & A_5 & A_6 & \ldots \\
\end{array}
\end{align*}

\textbf{Exceptionals}

\begin{align*}
\begin{array}{cccccccc}
E_5 & E_5/3= (E_5)^c & E_9 & E_9/3= (E_9)^c & (A\phi 3)^t & (A\phi 3)^{tc} & \ldots \\
\end{array}
\end{align*}

\textbf{Figure 2.} Classification of modules and subgroups of quantum SU(3).
SU(4)_k

Orbifold series

\[ A_1 \quad A_2 \quad A_3 \quad A_4 \quad \ldots \]

Conjugate orbifold series

\[ A_{1/2} \quad A_2^c \quad A_3^c \quad A_4^c \quad \ldots \]

\[ A_{1/2}^{[A]} \quad A_2^{\wedge} \quad A_3^{\wedge} \quad A_4^{\wedge} \quad \ldots \]

Figure 3
Figure 4. Classification of modules and subgroups of quantum $SU(4)$. 

Exceptionals

$E_4$  $E_6$  $E_6/5 = (E_6)^c$  $E_8^*$  $E_8/2 = (E_8)^c$  $(A_8/4)^c, (A_8/4)^{tc}$
Figure 5. Modules of exceptionals.
Figure 6. Exceptionally twisted modules of orbifolds.

\[ (SU(4))^{8/4}_t, (SU(4))^{8/4}_t, (SU(3))^{9/3}_t, (SU(3))^{9/3}_t, (SU(2))^{16/2}_t \]

Exceptionally twisted modules of orbifolds.
Figure 7. Modular ladder for the $D_5$ graph.
Figure 8. Modular ladder for the $E_6$ graph.
Figure 9. Modular ladder for the $E_7$ graph.
Figure 11. Modular ladder for the $E_8$ graph.