The classification of subgroups of quantum SU(N)

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I. The classification and structure of subgroups of quantum groups
I. The classification and structure of quantum subgroups of $\mathcal{S}_4$.

$\mathcal{S}_4$ and $\mathcal{S}_5$ respectively.

The quantum group $\mathcal{S}_4$ has subgroups and modules. Each subgroup and module in $\mathcal{S}_4$ has an associated quantum subgroup and module in $\mathcal{S}_5$. The minimal permutation of the higher-Coxeter group corresponds to $\mathcal{S}_4$.

2. The classification and structure of quantum subgroups of $\mathcal{S}_5$.

A quantum subgroup $\mathcal{S}_5$ of $\mathcal{S}_4$ is characterized by the property that the quantum subgroup $\mathcal{S}_4$ is a tensor product of two quantum subgroups of $\mathcal{S}_4$. The classification theorem for quantum subgroups of $\mathcal{S}_4$ states that any quantum subgroup of $\mathcal{S}_4$ is isomorphic to a tensor product of two quantum subgroups of $\mathcal{S}_4$. The classification theorem for quantum subgroups of $\mathcal{S}_5$ states that any quantum subgroup of $\mathcal{S}_5$ is isomorphic to a tensor product of two quantum subgroups of $\mathcal{S}_5$.
\[ \sum_{x} \left( \mathcal{F}^{(P)} \mathcal{W}^{(P)} \right) \]

where \( \mathcal{W} \) is a decomposition of a product.

Whereas the numbers \( x \) and \( y \) have no relationship to the model, we have shown that \( \mathcal{W} \) is a decomposition of a product.

We have developed methods for an exhaustive description of all possible solutions of the \( \mathcal{W} \) equation corresponding to the solution of the \( \mathcal{W} \) equation. The existence of these solutions is discussed in detail in the following sections.

The connection between the problem of the \( \mathcal{W} \) equation and the \( \mathcal{W} \) equation is a deep and complex one. The problem of the \( \mathcal{W} \) equation is a deep and complex one.

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The exceptional case of the non-ordinary version of the dual module.

We have therefore concluded the classification of non-ordinary exceptions:

1. The dual module for the non-ordinary case.

In the case of the non-ordinary version of the dual module, we have:

\[
\mathcal{D}^+(\mathcal{F}(\mathcal{G})) = \mathcal{D}^+(\mathcal{F}(\mathcal{G}))(\mathcal{W}) \leq \mathcal{D}^+(\mathcal{F}(\mathcal{G}))(\mathcal{W})
\]

where \(\mathcal{W}\) is the dual module for the non-ordinary version of the dual module.

We have therefore concluded the classification of non-ordinary exceptions.
where $|M| \geq \frac{\sqrt{N}}{\sqrt{\lambda - 1}}$. For $\lambda$ and $\alpha$ we denote by $\mathcal{F}$ the set of all the $\alpha$-tuples of real numbers that have

Fundamental Inequality:

and the fundamental principle. When the $\alpha$-tuple of real numbers has a nonzero counter, let $\mathbf{1}$ be the quantum dimension and $\mathbf{2}$ its

For a nonzero dimension, $\mathbf{1}$ denote by the quantum dimension, and let

orthogonal bases correspond to $\mathbf{0}$. To

principal of the denominator of the $\mathbf{0}$, of course, of $\mathbf{1}$? The latter case corresponds to the

devector. Therefore, the unique vector of $\mathbf{0}$, of course, of $\mathbf{1}$. We can show

For the interpretation of $\mathbf{2}$, the first of the quantum dimension is $\mathbf{0} + \lambda \mathbf{10}$

We start from the quantum splitting identity, which is nontrivial, and show that

We omit proofs. Here, $\mathbf{0}$, of course, of $\mathbf{1}$. However, the methods are

The nontrivial cases of the $\mathbf{2}$, of course, of $\mathbf{1}$. We can show

Suppose that $\mathbf{1}$ is any given rank, and let $\mathbf{2}$ be any present order.

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The nontrivial cases of the $\mathbf{2}$, of course, of $\mathbf{1}$. We can show

I.2. Rightly, effective bounds and construction algorithms for

do have are suggested for $\mathcal{F}$. I.2. Rightly, although the methods that we

II.3. In consequence the fact that there are sufficiently many graphs, and the

do not have effective general methods to construct the moduli of

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The problem, posed by Vaughan Jones, was to find a choice of formulas for the coefficients of the monomial form of the expression of the \(w\), expressed in terms of the former basis coefficients of monomials.

In general, a repeated application of the Weil transform formula above one

\[
1 \cdot \frac{d}{d} \left[ \frac{d}{d} \right] \frac{1}{[1 - u]} \frac{1 - u}{[1 - ud]} - \frac{1}{[1 - ud]} = u
\]

From which

\[1 \cdot \frac{1}{[1 - u]} \frac{1}{[1 - u]} = u\]

From this we obtain the recurrence relation

The Weil transform provides the recurrence relation

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\]
\[ (1 + y_1 + y_2 + \ldots + y_n) \prod_{i=1}^{n} \left( \frac{s^{y_i}}{i^{y_i}} \right) \leq \prod_{i=1}^{n} \left( 1 + y_i \right) \]

**Lemma:** Let \( \mathcal{G} \) be a planar graph, and let \( (\{t_1, t_2, \ldots, t_n\}) \subseteq V(\mathcal{G}) \). Then the following statement is true:

\[ \left| \mathcal{G}_1 \right| + \left| \mathcal{G}_2 \right| + \ldots + \left| \mathcal{G}_n \right| \leq \left| \mathcal{G} \right| \]

for every partition of the vertices into \( n \) subsets \( \mathcal{G}_1, \mathcal{G}_2, \ldots, \mathcal{G}_n \) of \( \mathcal{G} \).
The geometricization of quantum subgroups: a construction of weight lattices and roots from quantum subgroups. Subgroups have been built into a lattice of quantum subgroups, \( SU(2) \), and their structure is a quantum subgroup of \( SU(3) \). The bridge between these two structures is based on the following: the lattices of the quantum subgroups \( SU(2) \) and \( SU(3) \) are isomorphic to \( ADE \) graphs, and the \( ADE \) graphs are based on the root lattice of the Coxeter group. Hence, the structure of quantum subgroups is based on the Coxeter group, and this structure is related to the root lattice of the Coxeter group. The lattices of the quantum subgroups \( SU(2) \) and \( SU(3) \) are isomorphic to the Coxeter graphs of the Coxeter group, and the Coxeter graphs are based on the root lattice of the Coxeter group. Hence, the structure of quantum subgroups is based on the Coxeter group, and this structure is related to the root lattice of the Coxeter group.
Accordingly we combine computing the intersection of \( \sigma^N(x) / S \) of the leading edge of the diagram at \( x \) with the leading edge of the diagram at \( y \) to get the diagram at \( x \) as a consequence of the leading edge of the diagram at \( y \).

In order to compute the intersection of the leading edge of the diagram at \( x \) with the leading edge of the diagram at \( y \), we first compute the intersection of the leading edge of the diagram at \( x \) with the leading edge of the diagram at \( y \). We then compute the intersection of the leading edge of the diagram at \( x \) with the leading edge of the diagram at \( y \). Finally, we compute the intersection of the leading edge of the diagram at \( x \) with the leading edge of the diagram at \( y \).
The inner product is defined for each weight vector of the group G. All roots of a weight vector of the form of the weight vector of the group G.

\[(\varepsilon x)^{d-a}\cdot \frac{f}{(\varepsilon x)} = \langle \langle f, \Phi \rangle (\varepsilon x) \rangle \]

The weight vector is the integer valued homogeneous function on the weight of the group G. For any group G, the inner product of the weight vector of the group G can be defined by the formula of the weight vector of the group G.

\[\langle \langle f, \Phi \rangle (\varepsilon x) \rangle \]

We define the inner product between two weight vectors \(\varepsilon x\) and \(\eta y\) of the group G by the function \(\langle \langle f, \Phi \rangle (\varepsilon x) \rangle \).

\[\langle \langle f, \Phi \rangle (\varepsilon x) \rangle \]

The above formula for the inner product is due to the weight vector of the group G.

\[\langle \langle f, \Phi \rangle (\varepsilon x) \rangle \]

We define the inner product between two weight vectors \(\varepsilon x\) and \(\eta y\) of the group G by the formula of the weight vector of the group G.

\[\langle \langle f, \Phi \rangle (\varepsilon x) \rangle \]

The above formula for the inner product is due to the weight vector of the group G.
The classification of surjective linear maps of order $\gamma$ 

\[ \sum_{\gamma} (b)^{\gamma} I_{\gamma} = e^{\gamma} \circ \rho \]

Consider now extensions of $E$ by $F$. The linear spaces $E$ and $F$ are

\[ \text{Hom}(E, F) = \sum_{\gamma} (b)^{\gamma} I_{\gamma} \]

\[ \text{Ext}(E, F) = \bigoplus_{\gamma} (b)^{\gamma} I_{\gamma} \]

An important theorem in the classification of linear maps is the following:

\[ \text{Ext}(E, F) = \bigoplus_{\gamma} (b)^{\gamma} I_{\gamma} \]

The proof of this theorem is beyond the scope of this paper. For more details, please refer to the original source.

\[ \text{Ext}(E, F) = \bigoplus_{\gamma} (b)^{\gamma} I_{\gamma} \]

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In order to make such models of OTT feasible, which is a central problem in the field of quantum mechanics, there are some major challenges that need to be addressed. The first direction that should be emphasized in this context is the construction of the geometric framework. A geometric framework allows for the description of quantum states and their evolution, which is essential for understanding the behavior of quantum systems. We propose a new concept of geometric framework for quantum states.

2. New Directions of Research

(a) The geometric framework for quantum states
(b) The construction of geometric frameworks
(c) The development of geometric frameworks

These directions represent a significant advancement in the field of quantum mechanics and will provide a solid foundation for future research.

Appendix 1

A detailed discussion of the geometric framework for quantum states can be found in the appendix. Here, we briefly outline the main ideas and results.
In this paper, we discuss the phenomena of quantum entanglement. The vertices of a graph represent possible states, while the edges represent the transitions between these states. The quantum state of a system is described by a wave function, which is a superposition of all possible states.

**Vertices.** The vertices of the graphs are the possible configurations of the system.

**Edges.** The edges represent the evolution of the system between different configurations.

**Quantum Entanglement.** Quantum entanglement is a phenomenon where the states of two or more particles become correlated, such that the state of one particle cannot be described independently of the state of the other.

**Applications.** Quantum entanglement has applications in quantum computing, quantum cryptography, and quantum communication.

3. Conclusions

In conclusion, the study of quantum entanglement is now a fundamental aspect of modern physics.
The module structure is as follows:

\[ \text{\textbf{Modules}} \]

- **\text{\textbf{0}}** (the trivial module): Each module produces a trivial module, which is
- **\text{\textbf{1}}** (the identity module): Each module produces a module isomorphic to itself.

**Notation:**

- \( \Phi(A) \) for \( A \in \text{\textbf{Mod}} \): The endomorphism of \( A \).
- \( \text{End}(A) \): The endomorphism ring of \( A \).
- \( \text{Hom}(A, B) \): The set of homomorphisms from \( A \) to \( B \).

**Contravariant Inclusions:** All \( \phi \in \text{\textbf{Mod}} \) and \( \phi \in \text{\textbf{Mod}} \) give a contravariant inclusion.

**Oblivious bases and contravariant.**

**Exponentials.** For each \( N \), there is an exponential and a monoid.

**Quotient.** The quotient module \( M = N / \text{\textbf{Mod}} \) corresponds to a submonoid of \( N \).
REFERENCES

not all exceptional graphs come from cofinal inclusion

where the exceptional graphs show that

the same

incoherence for 2-dimensional

the graphs are in our classification of small index subgraphs. The

the exceptional graphs that remain after classifying by canonical.

The connected component of 0

the classification of subgroups of Quaternions (πn).
**SU(2)_k**

Orbifold series

\[
\begin{array}{cccccc}
A_2 & A_3 & A_4 & A_5 & D_4 & D_5 \\
(A_1)(A_2) & (A_3) & (A_4) & (A_4/2) & (A_6/2) & (A_8/2) \\
\end{array}
\]

\[
\begin{array}{cccc}
D_6 & D_7 \\
(A_10/2) & \ldots \\
\end{array}
\]

Exceptionals

\[
\begin{array}{lll}
E_6 & E_7 & E_8 \\
(E_{10}) & \left( (A_{16}/2)^t \right) & (E_{28}) \\
\end{array}
\]

*Figure 1.* Classification of modules and subgroups of quantum $SU(2)$. 
**SU(3)_k**

**Orbifold series**

\[ \begin{array}{cccccc}
A_1 & A_2 & A_3 & A_4 & A_5 & A_6 & \ldots \\
A_1/\beta & A_2/\beta & A_3/3 & A_4/\beta & A_5/\beta & A_6/3 & \ldots \\
\end{array} \]

**Conjugate orbifold series**

\[ \begin{array}{ccccccc}
A_1 \beta & A_2 \beta & A_3^c & A_4^c & A_5^c & A_6^c & \ldots \\
[3A_1^c][3A_2^c] & 3A_3^c & 3A_4^c & 3A_5^c & 3A_6^c & \ldots \\
\end{array} \]

**Exceptionals**

\[ \begin{array}{cccccc}
E_5 & E_5/3=(E_5)^c & E_9 & E_9/3=(E_9)^c & (A\beta^3)^t & (A\beta^3)^{tc} & E_{21} \\
\end{array} \]

Figure 2. Classification of modules and subgroups of quantum SU(3).
$\text{SU}(4)_k$

Orbifold series

\begin{align*}
A_1 & \quad A_2 & \quad A_3 & \quad A_4 & \quad \ldots \\
A_{1/2} & \quad A_{2/2} & \quad A_{3/2} & \quad A_{4/2} & \quad \ldots
\end{align*}

Conjugate orbifold series

\begin{align*}
A_{1/2} & \quad [\bar{A}_1] & \quad A_2^c & \quad A_3^c & \quad A_4^c & \quad \ldots \\
A_{1/2} & \quad [2A_1^c] & \quad A_{2/2}^c & \quad 2A_3^c & \quad 2A_4^c & \quad \ldots
\end{align*}

Figure 3
Figure 4. Classification of modules and subgroups of quantum $SU(4)$. 

Exceptionals

$E_4$  $E_6$  $E_6/5 = (E_6)^c$  $E_8$  $E_8/2 = (E_8)^c$  $(A_8/4)^t, (A_8/4)^{tc}$
Figure 5. Modules of exceptionals.

$E_6$  $E_8$  $E_6$  $E_6$  $E_6$

Modules of exceptionals

$SU(3)$  $SU(4)$  $SU(5)$  $SU(6)$
Figure 6. Exceptionally twisted modules of orbifolds.

\[
(\mathbb{Z}/2\mathbb{Z})^2 \leftrightarrow \{2,1,0\} \leftrightarrow \{2,2,2\} \leftrightarrow \{2,2,2\} \leftrightarrow \{1,1\} \leftrightarrow \{3,3\} \leftrightarrow \{3,3\} \leftrightarrow \{0,3\} \leftrightarrow \{3,0\}.
\]

\[
(\mathbb{Z}/4\mathbb{Z}) \leftrightarrow \{1,0\} \leftrightarrow \{2,1,0\} \leftrightarrow \{0,1,2\} \leftrightarrow \{1,1\} \leftrightarrow \{0,3\} \leftrightarrow \{3,0\}.
\]

Exceptionally twisted modules of orbifolds.
Figure 7. Modular ladder for the $D_5$ graph.
Figure 8. Modular ladder for the $E_6$ graph.
Figure 9. Modular ladder for the $E_7$ graph.
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**Figure 10**

The classification of subgroups of quantum SU(N)
\begin{figure}
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\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|}
\hline
1 & 14 & 15 & 16 & 17 & 18 & 19 & 20 & 21 \\
\hline
1 & (0,0) & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
2 & (1,0) & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
3 & (0,1) & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
4 & (2,0) & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
5 & (1,1) & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
6 & (0,2) & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
7 & (3,0) & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
8 & (2,1) & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
9 & (1,2) & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
10 & (0,3) & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
11 & (1,0) & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
12 & (2,0) & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
13 & (3,0) & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
14 & (1,0) & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
15 & (1,1) & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
16 & (1,7) & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
17 & (1,8) & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
18 & (1,9) & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
19 & (1,18) & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
20 & (1,20) & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
21 & (1,21) & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
22 & (1,22) & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
23 & (1,23) & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
24 & (1,24) & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
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\end{tabular}
\caption{Modular ladder for the $E_6$ graph.}
\end{figure}