The classification of subgroups of quantum SU(N)

Adrian Ocneanu

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is a matrix with entries \( A_{ij} \) for \( \{1, 2, \ldots, n\} \in \mathbb{R}^N \). The entries appear in several different forms in the theory: A number of important properties of the group \( (\mathbb{Z}/m\mathbb{Z})^n \) and its modules can be derived from the algebraic properties of \( \mathbb{Z}/m\mathbb{Z} \) and \( \mathbb{Z}/n\mathbb{Z} \), and of course from an exceptional basis of the algebraic properties of \( \mathbb{Z}/m\mathbb{Z} \); these properties make the group \( (\mathbb{Z}/m\mathbb{Z})^n \) and its modules more interesting in the theory of quantum groups. The modules which are not subject to coordinates in the sense of quantum groups, in addition to coordinates, have no more for which the quantum groups. In [1], we showed a new phenomenon that is very similar to the phenomenon of the quantum groups:

\[ (\mathbb{Z}/m\mathbb{Z})^n \]

The classification and structure of subgroups of quantum groups

1. The classification of subgroups of quantum groups

\[ (\mathbb{Z}/n\mathbb{Z})^N \]
1.3. The classification and structure of quaternion subgroups of $\text{SL}(3,\mathbb{R})$.

$\text{SL}(4,\mathbb{R})$ and $\text{SL}(5,\mathbb{R})$ respectively.

The quaternion group $\text{SL}(3,\mathbb{R})$ consists of the classification of subgroups of $\text{SL}(3,\mathbb{R})$.

A point in $\text{SL}(3,\mathbb{R})$ is an equivalence class of $3$-dimensional subspaces of $\mathbb{R}^3$. Two subspaces are equivalent if they have the same dimension and are linearly dependent. The classification of subspaces is given by the characteristic polynomial of the companion matrix.

The characteristic polynomial of a matrix $A$ is given by $\det(A - \lambda I) = 0$, where $I$ is the identity matrix and $\lambda$ is a complex number. The roots of this polynomial are the eigenvalues of $A$. The multiplicity of an eigenvalue is the number of times it appears as a root.

The characteristic polynomial of $A$ is $\det(A - \lambda I) = \lambda^3 - \text{tr}(A)\lambda^2 + \text{tr}(A\text{adj}(A))\lambda - \det(A)$, where $\text{tr}(A)$ is the trace of $A$ and $\text{adj}(A)$ is the adjugate of $A$.

The eigenvalues of $A$ are the roots of the characteristic polynomial. The eigenvectors associated with an eigenvalue are the non-zero solutions to $(A - \lambda I)v = 0$. The eigenspace of $A$ associated with $\lambda$ is the set of all eigenvectors corresponding to $\lambda$.

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the classification of structures of a graph \[ G \]

After completing the problem on the left side of the page, we have drawn the desired graph \( G \). Now, for any set of vertices \( S \), let the graph formed by the vertices not in \( S \) be denoted as \( G[V \setminus S] \). The problem here is to find any complete graph that contains \( G \) as a subgraph. For the classification, we need to develop conditions that facilitate the recognition of \( G \) as a complete graph. The problem of classifying the subgraphs and complete subgraphs of a graph \( G \) consists of common vertices and edges. The complete graph \( G \) is a family of graphs having as common vertices \( V(G) \) and edges \( E(G) \).
The exceptional cases of $\mathcal{E}(\mathcal{F})_{\mathcal{A}}$ or $\mathcal{E}(\mathcal{G})_{\mathcal{A}}$ or $\mathcal{E}(\mathcal{D})_{\mathcal{A}}$ arise because the $\mathcal{A}$-adic completion of the configuration $\mathcal{E}(\mathcal{F})_{\mathcal{A}}$ or $\mathcal{E}(\mathcal{G})_{\mathcal{A}}$ or $\mathcal{E}(\mathcal{D})_{\mathcal{A}}$ in general is not defined, and the number of exceptional cases is limited to the $\mathcal{A}$-adic completion of the configuration $\mathcal{E}(\mathcal{F})_{\mathcal{A}}$ or $\mathcal{E}(\mathcal{G})_{\mathcal{A}}$ or $\mathcal{E}(\mathcal{D})_{\mathcal{A}}$ in general.

I. The configuration of $\mathcal{E}(\mathcal{F})_{\mathcal{A}}$ for the non-exceptional cases

To $\mathcal{E}(\mathcal{F})_{\mathcal{A}}$ or $\mathcal{E}(\mathcal{G})_{\mathcal{A}}$ or $\mathcal{E}(\mathcal{D})_{\mathcal{A}}$ the number of exceptional cases appears to decrease from $\mathcal{E}(\mathcal{F})_{\mathcal{A}}$ or $\mathcal{E}(\mathcal{G})_{\mathcal{A}}$ or $\mathcal{E}(\mathcal{D})_{\mathcal{A}}$ to $\mathcal{E}(\mathcal{F})_{\mathcal{A}}$ or $\mathcal{E}(\mathcal{G})_{\mathcal{A}}$ or $\mathcal{E}(\mathcal{D})_{\mathcal{A}}$ in general. If $\mathcal{F}$ and $\mathcal{G}$ and $\mathcal{D}$ there are two exceptional cases, and the following configurations $\mathcal{F}$ and $\mathcal{G}$ and $\mathcal{D}$ are defined, the configurations $\mathcal{F}$ and $\mathcal{G}$ and $\mathcal{D}$ are also cases when $\mathcal{F}$ and $\mathcal{G}$ and $\mathcal{D}$ are the configuration of $\mathcal{F}$ and $\mathcal{G}$ and $\mathcal{D}$ in general. The configuration $\mathcal{F}$ and $\mathcal{G}$ and $\mathcal{D}$ on the common boundary of the configuration $\mathcal{F}$ and $\mathcal{G}$ and $\mathcal{D}$ is defined, and the configuration $\mathcal{F}$ and $\mathcal{G}$ and $\mathcal{D}$ are cases of the configuration $\mathcal{F}$ and $\mathcal{G}$ and $\mathcal{D}$ in general.

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For a finite-dimensional normed space $V$, we have

$$\|x\|^2 = \sum_{i=1}^{n} x_i^2.$$
The problem, posed by Glashow, now is to find a choice formula for the expression of the \( S \)-matrix.

The expression is: \( \frac{1}{2} \mathcal{F} [\mathcal{G}] \). 

And from this:

\[
1 - |\mathcal{G}| = d = 1
\]

The resolvent method begins by calculating:

\[
\left( A + B \right) \mathcal{G} = C
\]

Where \( A \) is the index of the \( \mathcal{G} \) matrix, \( B \) is the index of the \( \mathcal{G} \) matrix, and \( C \) is the index of the \( \mathcal{G} \) matrix.

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The expression is:

\[
\mathcal{F} [\mathcal{G}] = \left( \mathcal{G} + \mathcal{H} \right) \mathcal{G}
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And from this:

\[
1 - |\mathcal{G}| = d = 1
\]
If the process of creating and annihilating the particles is non-dissipative, then the process of creation and annihilation is non-dissipative, and we can use the following expression for the weight of the planar graph $G$:

$$W^{\mathcal{L}(\mathcal{A},\mathcal{B},\mathcal{C})} = \prod_{i \in \mathcal{L}(\mathcal{A},\mathcal{B},\mathcal{C})} (1 + \frac{1}{2} \mathcal{A}_i \mathcal{B}_i \mathcal{C}_i)$$

where $\mathcal{A}_i$, $\mathcal{B}_i$, and $\mathcal{C}_i$ are the creation, annihilation, and creation operators, respectively, acting on the $i$-th particle. The product is taken over all particles in the system.
In a coordinate system, we have obtained a very efficient and natural construction of a canonical basis for the space of Hamiltonian $V_\gamma$-graphs. This construction allows us to recover the results of previous works on $V_\gamma$-graphs, and to extend them to a wider class of systems. The main idea is to identify a Hamiltonian $V_\gamma$-graph with a certain $\Gamma$-graph, and then to use the properties of this correspondence to find a Hamiltonian $V_\gamma$-graph from a given $\Gamma$-graph.

The construction of a Hamiltonian $V_\gamma$-graph starts from the Hamiltonian of the system, which is given by $H = \sum \gamma_i \phi_i^2$, where $\gamma_i$ are the coupling constants and $\phi_i$ are the field operators. The Hamiltonian $V_\gamma$-graph is then constructed by identifying each vertex with a $\gamma$-graph, and each edge with a coupling constant $\gamma_i$.

The central idea of the construction is to use the correspondence between $\Gamma$-graphs and Hamiltonian $V_\gamma$-graphs to find a Hamiltonian $V_\gamma$-graph from a given $\Gamma$-graph. This correspondence is given by the following correspondence:

- Each vertex of the $\Gamma$-graph corresponds to a Hamiltonian $V_\gamma$-graph.
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Accordingly we combine computing the interior of the structure to the module structure of the vector of algebras is $[f_0^2 \varphi, \omega]$. The algebra $\mathcal{E}$ of essential operations with the interior of essential operations correspond to the structure of $\mathbb{F}(x,y)$.

If we take the vertices of $\mathcal{G}$ as permutations of a support of $\mathcal{G}$,

$$
\{1 - u, \cdots, 1\} = y \text{ and for } \ell \in \{d\}, \theta \in \mathcal{E}_0 \text{ of } \mathcal{E}_0 \text{ a Jones projection, define the essential part of}
$$

and the interior is in order, and is otherwise. Then $(1 + \gamma)\mathcal{E}(y)\mathcal{E}(y)\mathcal{E}(y) = (\mathcal{E}(y)\mathcal{E}(y))\mathcal{E}(y)$ is equal to $\mathcal{E}(y)\mathcal{E}(y)$ in which the factors of essential operations in $\mathcal{G}$.

$$
\delta (\mathcal{E}(y)\mathcal{E}(y)) = \mathcal{E}(y) \text{ a path for a path but the}
$$

and $\mathcal{E}(y) > \varrho$, $\psi \neq \varphi$ be different positions. Now for $u = \ell$, $\mathcal{E}(y) \in \mathcal{E}(y)$.

We shall in the next define the $\mathcal{G}$ as the set of vertices.

We define the $\mathcal{G}$ as the number of $\mathcal{G}$ associated to the $\mathcal{G}$.

The $\mathcal{G}$ are now defined $\mathcal{G}$ which is the number of the $\mathcal{G}$.

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The most nontrivial row of the latter corresponds to $\mathbf{Z}$,
where $\mathbf{Z}$ is a graph with $\mathbf{Z}$ vertices.

The formula for the inner product between rows becomes, in the limit of the

We then conclude that the above limit corresponds to the

To simplify the notation, let's denote $\mathbf{Z}$ as the graph with $\mathbf{Z}$ vertices.

We then define the inner product between rows as follows. For each vertex $v \in \mathbf{Z}$,

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where $M$ is the weight of each row of the graph $\mathbf{Z}$.

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The classification of groups of matrices (SL_2(F))

\[ \exp(b)^{E_{\mathfrak{g}}} \cdot \exp(c)^{E_{\mathfrak{g}}} = \exp(c^g \cdot b)^{E_{\mathfrak{g}}} \]

The product of a number (b)\exp^{E_{\mathfrak{g}}} \cdot c \exp^{E_{\mathfrak{g}}} \text{ is defined similarly for \mathfrak{g} from } \mathfrak{g}. 

\( f \text{ is a vector in } \mathfrak{g}, \text{ define } \exp(f) \text{ by } (\exp(f) x)^t = \exp(f) \cdot x = \exp(f) E_{\mathfrak{g}} \cdot x. \)

\( E_{\mathfrak{g}} \) is an exponential map from \mathfrak{g} to \mathfrak{h}.

One can define the canonical basis of the dimension of the algebraic variety of \mathfrak{g}.

Homogeneous spaces are determined by the set of any two-dimensional subspaces.

The classification of groups of matrices (SL_2(F)) beyond commuting and commutative algebraic varieties of the fundamental group, as is in the case of \mathfrak{g}.
In order to make use of models of O(2) physics, which is a genuine problem in the case of genuine physics, a structure similar to the O(2) models would be a genuine problem.

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Two New Directions of Research

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...the study of simple Lie algebras and their representations.

3. Conclusions
on the other hand (the elements under the operation $\otimes$ form a group under multiplication).

### Modular Invariants

Each modular invariant $f$ is a $\psi$-invariant with $f(a) = \phi(a(f))$. For a fixed $\phi$, the set of modular invariants forms a group under composition.

### Contractions of Modular Invariants

- **Normal Inclusions**: All contractions of modular invariants are normal.
- **Exceptional Inclusions**: For each $N$, there is an exceptional extension.

### Exceptional Àlaheges

1. $\langle f \rangle$, the set of $\psi$-invariants, is a normal subgroup of $\langle f \rangle$.
2. $\langle f \rangle$ is generated by $\psi$.
3. $\langle f \rangle$ is a normal subgroup of $\langle f \rangle$.
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References

Not all exceptional graphs come from cofinal subgraphs. The classification showed that
the SL(3) and SL(4) classification showed that different graphs share the same
natural orientation of the graphs is part of one maximal class, which is
connection between graphs and moduli invariants was an open problem. The
a subset of the exceptional graphs by d'Inverno and Zuber. The graphs are
SL(2) modular invariants were classified by Cahn. The graphs for SL(3)
are the best that the classification of SL(2) and SL(3) are defined on the curves.
SU(2)\textsubscript{k}

Orbifold series

\[ \begin{array}{ccccccc}
* & * & * & * & * & & \\ \\
A_2 & A_3 & A_4 & A_5 & D_4 & D_5 & D_6 & D_7 \\
(A_1)(A_2) & (A_3) & (A_4) & (A_4/2) & (A_6/2) & (A_8/2) & (A_{10}/2) & \ldots
\end{array} \]

Exceptionals

\[ \begin{array}{ccc}
* & * & * \\
E_6 & E_7 & E_8 \\
(E_{10}) & ((A_{16}/2)') & (E_{28})
\end{array} \]

Figure 1. Classification of modules and subgroups of quantum SU(2).
**SU(3)\textsubscript{k}**

Orbifold series

<table>
<thead>
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<th>A_1</th>
<th>A_2</th>
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Conjugate orbifold series

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Exceptionals

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<th>E_5</th>
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<th>E_9</th>
<th>E_9/3 = (E_9)^c</th>
<th>(A\emptyset 3)^T</th>
<th>(A\emptyset 3)^{Tc}</th>
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**Figure 2.** Classification of modules and subgroups of quantum SU(3).
**SU(4)_k**

**Orbifold series**

\[ A_1 \quad A_2 \quad A_3 \quad A_4 \quad \ldots \]

\[ A_{1/2} \quad A_{2/2} \quad A_{3/2} \quad A_{4/2} \quad \ldots \]

**Conjugate orbifold series**

\[ A_{1/2} \quad A_2^c \quad A_3^c \quad A_4^c \quad \ldots \]

\[ A_1 \quad [A_1] \quad [A_2^c] \quad 2A_3^c \quad 2A_4^c \quad \ldots \]

**Figure 3**
Figure 4. Classification of modules and subgroups of quantum $SU(4)$. 

$\begin{align*}
A_2/4 & \quad A_6/4 & \quad A_8/4 & \quad \ldots & \quad A_2 \quad \left[2(A_2^c/2)\right] & \quad 2(A_6^c/2) & \quad 2(A_8^c/2) & \quad \ldots \\
\text{Exceptionals} & \\
E_4 & E_6 & E_8/5=(E_6)^c & E_8^c & E_8/2=(E_8)^c \quad (A_8/4)^t \quad (A_8/4)^{tc}$
\end{align*}$
Figure 5. Modules of exceptional

Modules of exceptions:

\( E_6 \)

\( E_8 \)

\( E_9 \)

\( E_5 \)
FIGURE 6. Exceptionally twisted modules of orbifolds.
Figure 7. Modular ladder for the $D_5$ graph.
Figure 8. Modular ladder for the $E_6$ graph.
Figure 9. Modular ladder for the $E_7$ graph.
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**Figure 10**
Figure 11. Modular ladder for the $E_8$ graph.