The classification of subgroups of quantum SU(N)
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The classification and structure of subgroups of quantum groups

Chapter One

The classification of subgroups of quantum groups
1.3. The classification and structure of automorphic subgroups of $\mathcal{S}(\mathcal{L})$.

$\mathcal{S}(\mathcal{L})$ and $\mathcal{S}(\mathcal{L})$ respectively represent the compact and non-compact subgroups of the automorphism group of $\mathcal{L}$. The ADP group is a subgroup of the automorphism group of $\mathcal{L}$ and is defined as the set of all automorphisms of $\mathcal{L}$ that satisfy certain properties.

Consider the case where the ADP group is a subgroup of $\mathcal{L}$. The problem of classifying and understanding the structure of automorphic subgroups of $\mathcal{S}(\mathcal{L})$ is a challenging task. One approach to solving this problem is to study the representations of the ADP group.

In the following sections, we will explore the classification and structure of automorphic subgroups of $\mathcal{S}(\mathcal{L})$. We will consider the action of the ADP group on the set of automorphic subgroups, and study the properties of these subgroups through the lens of representation theory.

1.2. Zhubei's higher order congruence problem. The theoretical framework for solving this problem involves the study of congruences between certain algebraic structures. The key concepts in this problem include congruence relations, modular arithmetic, and the action of the ADP group on the set of automorphic subgroups.

We have shown that the class $\mathcal{C}(\mathcal{L})$ of the ADP group contains all automorphic subgroups of $\mathcal{L}$, and that the modular invariant subgroups associated to these subgroups are congruence classes of elements in $\mathcal{S}(\mathcal{L})$. This provides a framework for understanding the structure of automorphic subgroups of $\mathcal{S}(\mathcal{L})$.
The classification of structures of quantum (\(\mathcal{G}V\)).

...
The exceptional groups of type rank 2 and \( SL(2) \) are the simplest examples of exceptional groups. The classification of the exceptional groups is based on the study of simple Lie algebras and their representations.

The exceptional groups are of several types, including:

1. Exceptional Lie algebras:
   - \( E_6 \)
   - \( E_7 \)
   - \( E_8 \)
   - \( F_4 \)
   - \( G_2 \)

2. Exceptional groups of type rank 2:
   - \( SL(2) \)

The classification of these groups is important in the theory of Lie algebras and representation theory.

The exceptional Lie algebras are connected to the exceptional groups of Lie type, which are finite subgroups of the exceptional Lie groups.

The classification of the exceptional groups is based on the study of the root systems and the Weyl groups associated with these algebras.

For example, the group \( E_8 \) is the largest of the exceptional groups, and it is related to the Monster group in the theory of finite simple groups.

The study of these groups has applications in various areas of mathematics, including algebra, geometry, and physics.
the GAP, we have for $\lambda$ and $\mu$ denote by $p_{\lambda\mu}$ the rest of the Young diagrams of real less than half

fundamental inequality:

for some diagram $\lambda$ denote by $p_{\lambda\mu}$ the quantum dimension and by $\ell_{\lambda}$

orthonormal basis

backward, except in the $S(3)$ case, a straightforward point. For the methods are

however, bound on error bounds. Nay such a basis. When these methods are

comparable above the maximum height. When this number

the backward decrease in the maximum height. These errors and equal to $\ell_{(\alpha)}$ for

are those positive, which is the number

We can now show that any given point in the significant of $S(\lambda)$ or $S(\lambda',\lambda''\ldots)$

Furthermore, for any high $\alpha$, the construction of $S(\lambda)$ is done, the methods are

For the purpose of the reader, we assume that $S(\lambda)$ does not come from a quantum

$S(\lambda)$, and $S(\lambda',\lambda''\ldots)$ for

the torsion of $S(\lambda)$, and $S(\lambda',\lambda''\ldots)$ for

1.5 Rightly, effective bounds and construction algorithms for sub-

$\mathcal{A}$ where $\mathcal{A}$ the reader
The problem, stated by Vahlen Jones, was to find a choice formula for the coefficients of each monomial term in the expression of the denominator.

Given a product formula, with n factors:

\[ \prod_{i=1}^{n} \left( z_i + \alpha_i \right) \]

The coefficient of the term \( z_1^{a_1} \cdot z_2^{a_2} \cdot \ldots \cdot z_n^{a_n} \) in the expansion is given by:

\[ \prod_{i=1}^{n} \frac{\alpha_i^{a_i}}{a_i!} \]

For a product of the form:

\[ \prod_{i=1}^{n} \left( \beta_i + \gamma_i \right) \]

The coefficient of the term \( \beta_1^{b_1} \cdot \gamma_1^{c_1} \cdot \beta_2^{b_2} \cdot \gamma_2^{c_2} \cdot \ldots \cdot \beta_n^{b_n} \cdot \gamma_n^{c_n} \) in the expansion is given by:

\[ \prod_{i=1}^{n} \frac{\beta_i^{b_i} \gamma_i^{c_i}}{b_i! c_i!} \]

The problem is to find a formula for the coefficients of each monomial term in the expansion of the denominator.

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The coefficient of the term \( \beta_1^{b_1} \cdot \gamma_1^{c_1} \cdot \beta_2^{b_2} \cdot \gamma_2^{c_2} \cdot \ldots \cdot \beta_n^{b_n} \cdot \gamma_n^{c_n} \) in the expansion is given by:

\[ \prod_{i=1}^{n} \frac{\beta_i^{b_i} \gamma_i^{c_i}}{b_i! c_i!} \]
\[ (1 + \nu s + \nu^{-1} s) f \left( \prod_{1=\nu \mu \delta} \mathbf{S} \right) = (1, \nu, \mu, \delta) f \]
The quantum superpositions, a construction of the geometric realization of
quantum mechanics, are derived by drawing the quantum evolution of a
superposition. This evolution is represented by a series of unitary operators,
which evolve the state of the system through time. The unitary operators
are represented by matrices, and the evolution is given by the product of
these matrices.

In the geometric realization, the superposition is represented by a vector
in a Hilbert space, and the evolution is given by the action of a
time-dependent Hamiltonian. The Hamiltonian is a Hermitian operator,
which represents the energy of the system.

The geometric realization is used to study the dynamics of quantum systems,
and to derive the Schrödinger equation, which describes the evolution of
the wavefunction. The Schrödinger equation is a partial differential
equation, which describes the time evolution of the probability density of
the system.

The geometric realization is also used to study the entanglement of quantum
systems, and to derive the entanglement entropy, which is a measure of
the amount of entanglement.

The geometric realization is also used to study the quantum entanglement
in multipartite systems, and to derive the entanglement negativity, which is
a measure of the quantum entanglement in such systems.

The geometric realization is also used to study the quantum coherence,
and to derive the coherence measure, which is a measure of the coherence
of a quantum state.

The geometric realization is also used to study the quantum discord,
and to derive the quantum discord measure, which is a measure of the
quantum discord of a quantum state.

The geometric realization is also used to study the quantum correlations,
and to derive the quantum correlation measure, which is a measure of
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The geometric realization is also used to study the quantum information,
and to derive the quantum information measure, which is a measure of
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The geometric realization is also used to study the quantum error correction,
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The geometric realization is also used to study the quantum cryptography,
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The geometric realization is also used to study the quantum control,
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and to derive the quantum feedback measure, which is a measure of
the quantum feedback of a quantum state.
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\delta_{(w)}\Pi\bigg|_{\Pi} & \text{if } \delta_{(w)}\Pi\bigg|_{\Pi} \text{ is a non-empty product. Define the product}\n\ast_{\Pi} & \text{where } \Pi \text{ is a non-empty product.}
\end{cases}
\]

and (\Pi) = \Pi \text{ a non-empty product. Define the product}\n
\[
\bigg[\begin{pmatrix} \Pi \end{pmatrix} = (\Pi')^u \text{ for any } \in (d')^u : \delta_{(w)}\Pi\in d = \delta_{(w)}\Pi\bigg|_{\Pi} = \begin{cases}
\delta_{(w)}\Pi\bigg|_{\Pi} & \text{if } \delta_{(w)}\Pi\bigg|_{\Pi} \text{ is a non-empty product. Define the product}\n\ast_{\Pi} & \text{where } \Pi \text{ is a non-empty product.}
\end{cases}
\]

We may define the roots as the vertices of 1. We now define the roots of 1. We now define the roots of 1. We now define the roots of 1. We now define the roots of 1. We now define the roots of 1. We now define the roots of 1. We now define the roots of 1. We now define the roots of 1. We now define the roots of 1. We now define the roots of 1. We now define the roots of 1.
The theorem that the inner product between these vectors is zero unless they are the non-degenerate graph is easy to prove, as follows. For vector \( f \in \mathcal{H} \), we define the inner product between two vectors \( \langle f, g \rangle \) by

\[
\langle f, g \rangle = \sum_{n} a_n \overline{b_n},
\]

where \( A = \{a_n\} \) and \( B = \{b_n\} \) are the coefficients of \( f \) and \( g \). When \( \langle f, f \rangle \neq 0 \), we say that \( f \) is a non-degenerate vector.

We will use the following lemma.

**Lemma.** \( \langle f, g \rangle = 0 \) if and only if \( f \perp g \).

Proof. Suppose \( \langle f, g \rangle = 0 \). Then, for all \( h \in \mathcal{H} \),

\[
\langle h, f \rangle \langle g, h \rangle = \sum_{n} a_n \overline{b_n} = 0.
\]

Since \( f \perp g \), we have \( \langle f, h \rangle = 0 \) for all \( h \in \mathcal{H} \), so \( \langle f, g \rangle = 0 \) implies \( \langle f, h \rangle = 0 \) for all \( h \in \mathcal{H} \).

Conversely, suppose \( \langle f, g \rangle = 0 \). Then, for all \( h \in \mathcal{H} \),

\[
\langle h, f \rangle \langle g, h \rangle = \sum_{n} a_n \overline{b_n} = 0.
\]

Since \( f \perp g \), we have \( \langle f, h \rangle = 0 \) for all \( h \in \mathcal{H} \), so \( \langle f, g \rangle = 0 \) implies \( \langle f, h \rangle = 0 \) for all \( h \in \mathcal{H} \).

Therefore, \( \langle f, g \rangle = 0 \) if and only if \( f \perp g \).
\( \text{Hom}_{\mathcal{C}}(\mathcal{D}, \mathcal{F}) \otimes \delta \left( \mathcal{F}^\mathcal{H}(x) \right) \otimes \mathcal{F}(x, y) \mathcal{F} \oplus \mathcal{F}(y, x) \mathcal{F} \)
The conversion of higher dimensional QFT models, important for higher dimensional
emergent theories, raises several questions. The next section addresses the implications of these questions, which can then be expanded in the context of the full theory of
emergent gravity. The main results of this section can be summarized as follows:

1. The emergence of higher dimensional QFTs is strongly related to the emergence of
higher dimensional emergent theories.

2. New directions of research

A few of the main findings are highlighted in this section:

- The simplicity of the emergent QFTs
- The simplicity of the emergent QFTs
- The simplicity of the emergent QFTs

These findings suggest that the simplicity of the emergent QFTs

We have developed a very simple and novel way of constructing

These new QFTs, the simpler QFTs, and the full theory of the

These new QFTs, the simpler QFTs, and the full theory of the
The classification of structures of quantum systems depends on the nature of quantum structures and their interactions. Quantum structures are described by quantum mechanics, which studies the behavior of quantum systems under various conditions. The quantum structures can be classified into different categories based on their properties and interactions.

1. Introduction

2. Structures

3. Conclusions

4. References

In summary, the classification of quantum systems is essential for understanding the behavior of quantum structures under various conditions. The classification of quantum systems helps in the development of new technologies and applications in fields such as quantum computing and quantum mechanics.
The module structure of $\mathcal{A}(\mathcal{N})$ is a manifestation of the structure of the module $\mathcal{A}(\mathcal{N})$. Each module produces a module structure, which is

\[ (\mathcal{A}(\mathcal{N}))_{\mathcal{M}} \quad \text{for} \quad \mathcal{M} \in \mathcal{A}(\mathcal{N}). \]

The module structure of $\mathcal{A}(\mathcal{N})_{\mathcal{M}}$ is a manifestation of the structure of the module $\mathcal{A}(\mathcal{N})_{\mathcal{M}}$. Each module produces a module structure, which is

\[ (\mathcal{A}(\mathcal{N})_{\mathcal{M}})_{\mathcal{N}} \quad \text{for} \quad \mathcal{N} \in \mathcal{A}(\mathcal{N})_{\mathcal{M}}. \]

The module structure of $\mathcal{A}(\mathcal{N})_{\mathcal{M}}_{\mathcal{N}}$ is a manifestation of the structure of the module $\mathcal{A}(\mathcal{N})_{\mathcal{M}}_{\mathcal{N}}$. Each module produces a module structure, which is

\[ (\mathcal{A}(\mathcal{N})_{\mathcal{M}}_{\mathcal{N}})_{\mathcal{O}} \quad \text{for} \quad \mathcal{O} \in \mathcal{A}(\mathcal{N})_{\mathcal{M}}_{\mathcal{N}}. \]

The module structure of $\mathcal{A}(\mathcal{N})_{\mathcal{M}}_{\mathcal{N}}_{\mathcal{O}}$ is a manifestation of the structure of the module $\mathcal{A}(\mathcal{N})_{\mathcal{M}}_{\mathcal{N}}_{\mathcal{O}}$. Each module produces a module structure, which is

\[ (\mathcal{A}(\mathcal{N})_{\mathcal{M}}_{\mathcal{N}}_{\mathcal{O}})_{\mathcal{P}} \quad \text{for} \quad \mathcal{P} \in \mathcal{A}(\mathcal{N})_{\mathcal{M}}_{\mathcal{N}}_{\mathcal{O}}. \]

The module structure of $\mathcal{A}(\mathcal{N})_{\mathcal{M}}_{\mathcal{N}}_{\mathcal{O}}_{\mathcal{P}}$ is a manifestation of the structure of the module $\mathcal{A}(\mathcal{N})_{\mathcal{M}}_{\mathcal{N}}_{\mathcal{O}}_{\mathcal{P}}$. Each module produces a module structure, which is

\[ (\mathcal{A}(\mathcal{N})_{\mathcal{M}}_{\mathcal{N}}_{\mathcal{O}}_{\mathcal{P}})_{\mathcal{Q}} \quad \text{for} \quad \mathcal{Q} \in \mathcal{A}(\mathcal{N})_{\mathcal{M}}_{\mathcal{N}}_{\mathcal{O}}_{\mathcal{P}}. \]

The module structure of $\mathcal{A}(\mathcal{N})_{\mathcal{M}}_{\mathcal{N}}_{\mathcal{O}}_{\mathcal{P}}_{\mathcal{Q}}$ is a manifestation of the structure of the module $\mathcal{A}(\mathcal{N})_{\mathcal{M}}_{\mathcal{N}}_{\mathcal{O}}_{\mathcal{P}}_{\mathcal{Q}}$. Each module produces a module structure, which is

\[ (\mathcal{A}(\mathcal{N})_{\mathcal{M}}_{\mathcal{N}}_{\mathcal{O}}_{\mathcal{P}}_{\mathcal{Q}})_{\mathcal{R}} \quad \text{for} \quad \mathcal{R} \in \mathcal{A}(\mathcal{N})_{\mathcal{M}}_{\mathcal{N}}_{\mathcal{O}}_{\mathcal{P}}_{\mathcal{Q}}. \]

The module structure of $\mathcal{A}(\mathcal{N})_{\mathcal{M}}_{\mathcal{N}}_{\mathcal{O}}_{\mathcal{P}}_{\mathcal{Q}}_{\mathcal{R}}$ is a manifestation of the structure of the module $\mathcal{A}(\mathcal{N})_{\mathcal{M}}_{\mathcal{N}}_{\mathcal{O}}_{\mathcal{P}}_{\mathcal{Q}}_{\mathcal{R}}$. Each module produces a module structure, which is

\[ (\mathcal{A}(\mathcal{N})_{\mathcal{M}}_{\mathcal{N}}_{\mathcal{O}}_{\mathcal{P}}_{\mathcal{Q}}_{\mathcal{R}})_{\mathcal{S}} \quad \text{for} \quad \mathcal{S} \in \mathcal{A}(\mathcal{N})_{\mathcal{M}}_{\mathcal{N}}_{\mathcal{O}}_{\mathcal{P}}_{\mathcal{Q}}_{\mathcal{R}}. \]

The module structure of $\mathcal{A}(\mathcal{N})_{\mathcal{M}}_{\mathcal{N}}_{\mathcal{O}}_{\mathcal{P}}_{\mathcal{Q}}_{\mathcal{R}}_{\mathcal{S}}$ is a manifestation of the structure of the module $\mathcal{A}(\mathcal{N})_{\mathcal{M}}_{\mathcal{N}}_{\mathcal{O}}_{\mathcal{P}}_{\mathcal{Q}}_{\mathcal{R}}_{\mathcal{S}}$. Each module produces a module structure, which is

\[ (\mathcal{A}(\mathcal{N})_{\mathcal{M}}_{\mathcal{N}}_{\mathcal{O}}_{\mathcal{P}}_{\mathcal{Q}}_{\mathcal{R}}_{\mathcal{S}})_{\mathcal{T}} \quad \text{for} \quad \mathcal{T} \in \mathcal{A}(\mathcal{N})_{\mathcal{M}}_{\mathcal{N}}_{\mathcal{O}}_{\mathcal{P}}_{\mathcal{Q}}_{\mathcal{R}}_{\mathcal{S}}. \]
References
not all exceptional graphs come from contractible
modular invariants, and the exceptional $\mathfrak{L}$ of the $\mathfrak{L}$ classification shows that
$\mathfrak{T}_0^{\mathfrak{L}}$. The $\mathfrak{L}$ classification shows that different graphs can have the same
natural invariants of the graphs is part of our maximal admissible theory, which is
a subclass of the class proposed empirically by d'Inorosco and where we have
$\mathfrak{S}/\mathfrak{L}$ of the graphs, or the maximal admissible class by Cameron. The graphs for $\mathfrak{S}/\mathfrak{L}$
$\mathfrak{T}_0^{\mathfrak{L}}$. The graphs for $\mathfrak{S}/\mathfrak{L}$ arise in our classification of small index subgraphs. The
modular invariants of the graphs are classified by $\mathfrak{S}/\mathfrak{L}$ and $\mathfrak{T}_0^{\mathfrak{L}}$, and are defined on the focus. The
the last time the classification $\mathfrak{T}_0^{\mathfrak{L}}$ and $\mathfrak{S}/\mathfrak{L}$ are defined on the focus. The
classification of small graphs, $\mathfrak{T}_0^{\mathfrak{L}}$, $\mathfrak{S}/\mathfrak{L}$ and $\mathfrak{T}_0^{\mathfrak{L}}$, as defined by Cameron's
and $\mathfrak{T}_0^{\mathfrak{L}}$, are defined on the focus. The

References
SU(2)_k

Orbifold series

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<thead>
<tr>
<th>A_2</th>
<th>A_3</th>
<th>A_4</th>
<th>A_5</th>
<th>D_4</th>
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Exceptionals

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<td>(E_10)</td>
<td>((A_16/2)^t)</td>
<td>(E_28)</td>
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Figure 1. Classification of modules and subgroups of quantum SU(2).
\[ \text{SU}(3)_k \]

Orbifold series

\[
\begin{array}{ccccccc}
A_1 & A_2 & A_3 & A_4 & A_5 & A_6 & \ldots \\
A_{1/3} & A_{2/3} & A_{3/3} & A_{4/3} & A_{5/3} & A_{6/3} & \ldots
\end{array}
\]

Conjugate orbifold series

\[
\begin{array}{ccccccc}
A_1^\beta & A_2^\beta & A_3^c & A_4^c & A_5^c & A_6^c & \ldots \\
[3A_1^c] & [3A_2^c] & 3A_3^c & 3A_4^c & 3A_5^c & 3A_6^c & \ldots
\end{array}
\]

Exceptionals

\[
\begin{array}{ccccccc}
E_5 & E_5/3=(E_5)^c & E_9 & E_9/3=(E_9)^c & (A\phi 3)^t & (A\phi 3)^{tc} & E_{21}
\end{array}
\]

**Figure 2.** Classification of modules and subgroups of quantum \( SU(3) \).
\( \text{SU}(4)_k \)

Orbifold series

\[
\begin{align*}
A_1 & \quad A_2 & \quad A_3 & \quad A_4 & \quad \ldots \\
A_1/2 & \quad A_2/2 & \quad A_3/2 & \quad A_4/2 & \quad \ldots
\end{align*}
\]

Conjugate orbifold series

\[
\begin{align*}
A_1/2 & \quad A_2 & \quad A_3 & \quad A_4 & \quad \ldots \\
[2A_1] & \quad [2A_2] & \quad 2A_3 & \quad 2A_4 & \quad \ldots
\end{align*}
\]

Figure 3
Figure 4. Classification of modules and subgroups of quantum $SU(4)$.
Figure 3. Modules of exceptionals.
Exceptionally twisted modules of orbifolds:

\{1,0,1\} \leftrightarrow \{0,4,0\}, 0

\{2,1,0\} \leftrightarrow \{2,2,2\}, 1

\{0,1,2\} \leftrightarrow \{2,2,2\}, 3

\{1,1\} \leftrightarrow \{3,3\}, 0

\{0,3\} \leftrightarrow \{3,0\}

\{2\} \leftrightarrow \{8\}, 0

\{2\} \leftrightarrow \{8\}, 1

\{16/2\} \leftrightarrow \{8\}, 1

\{\mathbb{E}_7\} \leftrightarrow \{4/2\}
Figure 7. Modular ladder for the $D_5$ graph.
Figure 8. Modular ladder for the $E_6$ graph.
Figure 9. Modular ladder for the $E_7$ graph.
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**Figure 11.** Modular ladder for the $E_8$ graph.