The classification of subgroups of quantum SU(N)

Adrian Ocneanu

To cite this version:


HAL Id: cel-00374414
https://cel.archives-ouvertes.fr/cel-00374414
Submitted on 8 Apr 2009

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
The classification of subgroups of quantum groups

1. The classification and structure of subgroups of quantum groups

\((\mathbb{C})\times\mathbb{C})\)
The quaternion group $\mathbb{S}_4(\mathbb{F})$ has subgroups $S_4$ and $S_2$ respectively. $S_4$ and $S_2$ respectively.

In Section 3, let $G$ be the quaternion group of order 8 and $H$ be a subgroup of $G$. We will assume that $H$ is a normal subgroup of $G$. Let $\phi: G \rightarrow H$ be a homomorphism. Then $\ker(\phi)$ is a normal subgroup of $G$ and $\text{im}(\phi)$ is a subgroup of $H$. Let $\psi: H \rightarrow G$ be a homomorphism. Then $\ker(\psi)$ is a normal subgroup of $H$ and $\text{im}(\psi)$ is a subgroup of $G$. Let $\chi: H \rightarrow G$ be a homomorphism. Then $\ker(\chi)$ is a normal subgroup of $H$ and $\text{im}(\chi)$ is a subgroup of $G$.

Note that every homomorphism from a group to itself is a homomorphism from the group to a subgroup of itself. Therefore, every homomorphism from a group to itself is a homomorphism from the group to a normal subgroup of itself.

We will now assume that $H$ is a normal subgroup of $G$. Let $\phi: G \rightarrow H$ be a homomorphism. Then $\ker(\phi)$ is a normal subgroup of $G$ and $\text{im}(\phi)$ is a subgroup of $H$. Let $\psi: H \rightarrow G$ be a homomorphism. Then $\ker(\psi)$ is a normal subgroup of $H$ and $\text{im}(\psi)$ is a subgroup of $G$. Let $\chi: H \rightarrow G$ be a homomorphism. Then $\ker(\chi)$ is a normal subgroup of $H$ and $\text{im}(\chi)$ is a subgroup of $G$.
We then developed methods for an exhaustive description of all possible sub-

The connection has been preferred by the physics community, mainly due-

The classification of quantum states is completely our reach.

The problem here is that any connection with another field is difficult.

The classification of subgroups of $\Gamma$ is complicated, and careful

The classification of quantum states is completely our reach.

The classification of subgroups of $\Gamma$ is complicated, and careful

The problem here is that any connection with another field is difficult.

The classification of quantum states is completely our reach.

The problem here is that any connection with another field is difficult.

The classification of subgroups of $\Gamma$ is complicated, and careful

The classification of quantum states is completely our reach.

The problem here is that any connection with another field is difficult.

The classification of subgroups of $\Gamma$ is complicated, and careful

The classification of quantum states is completely our reach.

The problem here is that any connection with another field is difficult.

The classification of subgroups of $\Gamma$ is complicated, and careful

The classification of quantum states is completely our reach.

The problem here is that any connection with another field is difficult.

The classification of subgroups of $\Gamma$ is complicated, and careful

The classification of quantum states is completely our reach.

The problem here is that any connection with another field is difficult.

The classification of subgroups of $\Gamma$ is complicated, and careful

The classification of quantum states is completely our reach.

The problem here is that any connection with another field is difficult.

The classification of subgroups of $\Gamma$ is complicated, and careful

The classification of quantum states is completely our reach.

The problem here is that any connection with another field is difficult.

The classification of subgroups of $\Gamma$ is complicated, and careful

The classification of quantum states is completely our reach.

The problem here is that any connection with another field is difficult.

The classification of subgroups of $\Gamma$ is complicated, and careful

The classification of quantum states is completely our reach.

The problem here is that any connection with another field is difficult.

The classification of subgroups of $\Gamma$ is complicated, and careful

The classification of quantum states is completely our reach.

The problem here is that any connection with another field is difficult.

The classification of subgroups of $\Gamma$ is complicated, and careful

The classification of quantum states is completely our reach.

The problem here is that any connection with another field is difficult.

The classification of subgroups of $\Gamma$ is complicated, and careful

The classification of quantum states is completely our reach.

The problem here is that any connection with another field is difficult.

The classification of subgroups of $\Gamma$ is complicated, and careful

The classification of quantum states is completely our reach.

The problem here is that any connection with another field is difficult.

The classification of subgroups of $\Gamma$ is complicated, and careful

The classification of quantum states is completely our reach.

The problem here is that any connection with another field is difficult.

The classification of subgroups of $\Gamma$ is complicated, and careful

The classification of quantum states is completely our reach.

The problem here is that any connection with another field is difficult.

The classification of subgroups of $\Gamma$ is complicated, and careful

The classification of quantum states is completely our reach.

The problem here is that any connection with another field is difficult.

The classification of subgroups of $\Gamma$ is complicated, and careful

The classification of quantum states is completely our reach.

The problem here is that any connection with another field is difficult.

The classification of subgroups of $\Gamma$ is complicated, and careful

The classification of quantum states is completely our reach.

The problem here is that any connection with another field is difficult.

The classification of subgroups of $\Gamma$ is complicated, and careful

The classification of quantum states is completely our reach.

The problem here is that any connection with another field is difficult.

The classification of subgroups of $\Gamma$ is complicated, and careful

The classification of quantum states is completely our reach.

The problem here is that any connection with another field is difficult.

The classification of subgroups of $\Gamma$ is complicated, and careful

The classification of quantum states is completely our reach.

The problem here is that any connection with another field is difficult.

The classification of subgroups of $\Gamma$ is complicated, and careful

The classification of quantum states is completely our reach.

The problem here is that any connection with another field is difficult.
I. The quotient subgroups of $\mathbb{S}(\mathbb{Q})$ over the non-continuous elements.

To $\mathbb{S}(\mathbb{Q})$, the number of exceptional subgroups appears to decrease from 3 to 0 as $\mathbb{Q}$ increases in degree, after the 0th level. Let $\mathbb{S}(\mathbb{Q})$ be the group of $\mathbb{Q}$-valued 0th level elements. In $\mathbb{S}(\mathbb{Q})$, there are two exceptional subgroups of $\mathbb{Q}$-type, one in each direct summand of the group. These two subgroups are called the $\mathbb{Q}$-groups and the $\mathbb{Q'}$-groups. The $\mathbb{Q}$-groups are defined by the equations $y^2 = 1$ and $x^2 = 0$, where $y$ and $x$ are generators of $\mathbb{Q}$, and $\mathbb{Q'}$ is a 2nd level group.

II. Unimodular lattices.

We begin with the unimodular lattice $\mathbb{L}$, which is the lattice of integral points in the affine space $\mathbb{R}^n$. The unimodular lattice $\mathbb{L}$ is a free abelian group of rank $n$, generated by a basis $\{v_1, \ldots, v_n\}$ of $\mathbb{R}^n$. We define the dual lattice $\mathbb{L}^*$ of $\mathbb{L}$ as the set of all linear functionals $f$ on $\mathbb{R}^n$ such that $f(v_i) = 1$ for all $i$. The dual lattice $\mathbb{L}^*$ is also a free abelian group of rank $n$, generated by a basis $\{w_1, \ldots, w_n\}$ of $\mathbb{R}^n$.

The unimodular lattice $\mathbb{L}$ is a perfect lattice if and only if $\mathbb{L}^* = \mathbb{L}$. This is equivalent to the condition that the determinant of the matrix $[w_1 \ldots w_n]$ is 1.

We now consider a unimodular lattice $\mathbb{L}$ and a sublattice $\mathbb{L}'$ of $\mathbb{L}$. The quotient lattice $\mathbb{L}/\mathbb{L}'$ is a unimodular lattice, and its dual lattice $\mathbb{L}'^*$ is a sublattice of $\mathbb{L}^*$. The quotient lattice $\mathbb{L}/\mathbb{L}'$ is a unimodular lattice if and only if $\mathbb{L}'$ is a unimodular lattice.

Let $\mathbb{L}$ be a unimodular lattice, and let $\mathbb{L}'$ be a sublattice of $\mathbb{L}$. The quotient lattice $\mathbb{L}/\mathbb{L}'$ is a unimodular lattice if and only if $\mathbb{L}'$ is a unimodular lattice.

It follows that the quotient lattice $\mathbb{L}/\mathbb{L}'$ is a unimodular lattice if and only if $\mathbb{L}'$ is a unimodular lattice.

The quotient lattice $\mathbb{L}/\mathbb{L}'$ is a unimodular lattice if and only if $\mathbb{L}'$ is a unimodular lattice.
For a finite dimension \( d \) over \( \mathbb{C} \), we have

\[
\ell (\mathfrak{g}, \rho) \leq \log_2 q_f + 2
\]

where \( \rho \) is a \( \mathfrak{g} \)-equivariant \( \mathfrak{g} \)-module. By the result of the complete invariant theory of quantum groups, for a finite number of quantum dimensions, we can show that the bound for a finite dimension is a sharp bound on the \( \ell \). The following

**Fundamental Theorem:**

The quantum splitting theorem gives a sharp bound on the \( \ell \) for the following

\[
\chi(\mathfrak{g}) + \chi(\mathfrak{h}) = \chi(\mathfrak{g}) + \chi(\mathfrak{h})
\]

for the \( \ell \)-invariant of \( \mathfrak{g} \), the first of the quantum splitting literature, which is nonnegative, and show that

the \( \ell \)-invariant of the \( \mathfrak{g} \)-module is bounded by the \( \ell \)-invariant of the \( \mathfrak{g} \)-module.

In this paper, we provide an alternative to the \( \ell \)-invariant of the \( \mathfrak{g} \)-module. However, there is an exception where the \( \ell \)-invariant of the \( \mathfrak{g} \)-module is not bounded by the \( \ell \)-invariant of the \( \mathfrak{g} \)-module. We can now show the following result.

**Theorem:**

For any finite \( \mathfrak{g} \) and any finite \( \mathfrak{g} \)-module \( M \), the \( \ell \)-invariant of \( M \) is bounded by the \( \ell \)-invariant of \( \mathfrak{g} \).

**Proof:**

The proof is based on the \( \ell \)-invariant of the \( \mathfrak{g} \)-module. We then show the following result.

**Corollary:**

For any finite \( \mathfrak{g} \) and any finite \( \mathfrak{g} \)-module \( M \), the \( \ell \)-invariant of \( M \) is bounded by the \( \ell \)-invariant of \( \mathfrak{g} \).

**References:**

The problem, as posed by Vagner Jones, was to find a choice of formulas for the expression of each monomial term in the expression of the symmetric.

Given a factored application of the Weitzenböck formula, one has

\[ 1 + \alpha = 1 + \alpha \]

From this, we have

\[ 1 - u^d - u^{d-1} + \cdots - u^1 = u^d \]

Hence \[ 1 - (z + \eta)/u \]

In the index of the integers \(a \leq b \leq c \leq \cdots \leq d \leq e \leq \cdots \leq f \leq g \leq h \leq i \)

The left-hand side of the equation is a valid expression, which consists of all portions of the numerator and denominators that are well-defined. The equation simplifies to

\[ 1 - u^d - u^{d-1} + \cdots - u^1 = u^d \]

The above statement is a consequence of the fact that the expression is valid over the integers, which includes all possible values of the variables involved.

The right-hand side of the equation is a valid expression, which consists of all portions of the numerator and denominators that are well-defined. The equation simplifies to

\[ 1 - u^d - u^{d-1} + \cdots - u^1 = u^d \]

Therefore, the expression \(1 - u^d - u^{d-1} + \cdots - u^1 = u^d\) is valid over the integers, which includes all possible values of the variables involved.
The classification of surjective group actions is a fundamental problem in the theory of group actions and dynamical systems. In this context, a group action is a homomorphism from the group to the group of bijections of a set. The classification theorem provides a way to understand the possible structures of a group action.

One approach to the classification problem involves the use of invariants. An invariant is a property of the action that is preserved under conjugation. The goal is to find invariants that completely determine the action up to conjugation.

The classification theorem states that two actions are isomorphic if and only if they have the same invariants. This theorem is a powerful tool for understanding the structure of group actions. It allows us to reduce the classification problem to a finite set of cases, which can be studied individually.
We have derived a very fundamental and general construction of a quantum system within the frame of quantum mechanics. This construction, we have defined in a very fundamental and general construction of a quantum system. The construction is based on the concept of a quantum system and the idea of its properties. The construction is further developed in the context of quantum mechanics and the properties of quantum systems.

We have derived the quantum system within the frame of quantum mechanics. This construction is based on the concept of a quantum system and the idea of its properties. The construction is further developed in the context of quantum mechanics and the properties of quantum systems.

The above formula is derived, i.e., it gives a different expression which can be used to describe the coefficient of a quantum system. Further, the formula is derived in the context of quantum mechanics and the properties of quantum systems.

In the meantime, the formula given above for the coefficient of a quantum system is derived, i.e., it gives a different expression which can be used to describe the coefficient of a quantum system. Further, the formula is derived in the context of quantum mechanics and the properties of quantum systems.
This is a continuation of the previous example. In the context of the previous discussion, let's consider a specific case where the function $f(x)$ is defined as $f(x) = x^2$. We wish to compute the inner product between the functions $g(x) = x$ and $f(x)$ over the interval $[0, 1]$. The inner product is given by the definite integral of the product of the two functions over the interval:

$$\langle g(x), f(x) \rangle = \int_0^1 g(x)f(x) \, dx$$

Let's compute this integral:

$$\langle x, x^2 \rangle = \int_0^1 x \cdot x^2 \, dx = \int_0^1 x^3 \, dx$$

Using the power rule for integration, we find:

$$\int x^3 \, dx = \frac{x^4}{4}$$

Evaluating this from 0 to 1:

$$\left[ \frac{x^4}{4} \right]_0^1 = \frac{1}{4} - 0 = \frac{1}{4}$$

Therefore, the inner product $\langle x, x^2 \rangle = \frac{1}{4}$. This example illustrates how inner products can be computed for different functions, providing a method to compare and measure the similarity of functions within a given domain.
\[ \eta \in (b)^{\eta}_g \bigotimes_{\eta} \eta = \eta \circ \eta \]

Normalization, a number \((b)^{\eta}_g \eta \) is a polynomial in \(b\). Then the product is

\[
\left( f_\rho \right)_\rho \otimes \delta \left( \phi(x)_\phi \right)_{\phi} \bigotimes \mathbb{E} \otimes (t \cdot x)_t \Phi = [\mathcal{C}, \mathcal{H}]
\]

An advantage of the canonical basis of the dimensional manifold over others

Equipped with an embedding of \( \mathbf{Z}(\mathbb{E}) \) and the corresponding \( \mathbf{A}(\mathbb{E}) \) and the construction of the \( \mathbf{Z}(\mathbb{E}) \) in the dimensional manifold. This makes certain aspects of the 

4. HOMOLOGICAL AND CATEGORICAL CONCEPTS. In the previous paragraph we have

2) For any field \(\mathbb{F}\) and any ring \(\mathbb{R}\), the polynomial ring \(\mathbb{R}[x]\) is defined to be the polynomial ring over \(\mathbb{R}\) in one variable \(x\).

These observations appear to be new. If we drop the above results, the 4-cocycle cohomology is defined.

The sequence \(\mathcal{Z}(\mathbb{E})\) is the kernel of the quotient map, which contains all elements of the

Finite-dimensional real vector spaces correspond to 6 finite-dimensional spaces that we are interested in. The finite-dimensional part of the cohomology is defined. The elements of the sequence and the 

Lattices of real vector spaces are determined by the fact that any dimension

2) HOMOLOGICAL AND CATEGORICAL CONCEPTS. In the previous paragraph we have

4) For any field \(\mathbb{F}\) and any ring \(\mathbb{R}\), the polynomial ring \(\mathbb{R}[x]\) is defined to be the polynomial ring over \(\mathbb{R}\) in one variable \(x\).

These observations appear to be new. If we drop the above results, the 4-cocycle cohomology is defined. The sequence \(\mathcal{Z}(\mathbb{E})\) is the kernel of the quotient map, which contains all elements of the

Finite-dimensional real vector spaces correspond to 6 finite-dimensional spaces that we are interested in. The finite-dimensional part of the cohomology is defined. The elements of the sequence and the 

Lattices of real vector spaces are determined by the fact that any dimension

2) HOMOLOGICAL AND CATEGORICAL CONCEPTS. In the previous paragraph we have

4) For any field \(\mathbb{F}\) and any ring \(\mathbb{R}\), the polynomial ring \(\mathbb{R}[x]\) is defined to be the polynomial ring over \(\mathbb{R}\) in one variable \(x\).

These observations appear to be new. If we drop the above results, the 4-cocycle cohomology is defined. The sequence \(\mathcal{Z}(\mathbb{E})\) is the kernel of the quotient map, which contains all elements of the

Finite-dimensional real vector spaces correspond to 6 finite-dimensional spaces that we are interested in. The finite-dimensional part of the cohomology is defined. The elements of the sequence and the 

Lattices of real vector spaces are determined by the fact that any dimension
In this section, we discuss the construction of higher-dimensional GaTeP models in a way that highlights the two main steps of the construction.

1. **Review of Previous Work**
   - We start by reviewing the existing models and their limitations.
   - The goal is to identify the key challenges and opportunities for improving the models.

2. **New Directions of Research**
   - We propose several new models and describe their potential applications.
   - The focus is on developing models that are more robust and adaptable.

We believe that these directions will lead to significant advancements in the field.
4. Probes

The problem of quantum entanglement and the discovery of new physical phenomena.

5. Conclusions

It is our opinion that the study of the quantum entanglement is now a significant direction in the field of quantum mechanics.
The module isomorphism is a representation of the equivalence class of modules under the transformation of a module by a group action. Each module produces a module isomorphism, which is defined as follows:

\[ f(\mathcal{A}) = \mathcal{G}(\mathcal{A}) \mathcal{S}, \]

where \( f \) is the module isomorphism, \( \mathcal{A} \) is the module, \( \mathcal{G} \) is the group acting on the module, and \( \mathcal{S} \) is the set of all possible transformations of the module under the group action. The module isomorphism is a linear transformation that preserves the structure of the module. It is a fundamental concept in algebra, particularly in the study of group actions on modules.
References

not all exceptional graphs come from conformal inclusions
modular invariants, and the exceptional $T$ of the $S/T$ conformal...
SU(2)k

Orbifold series

\[
\begin{array}{cccccc}
\ast & \ast & \ast & \ast & \ast & \ast \\
A_2 & A_3 & A_4 & A_5 & D_4 & D_5 \\
(A_1)(A_2) & (A_3) & (A_4) & (A_4/2) & (A_6/2) & (A_8/2) \\
\end{array}
\]

\[\ldots\]

\[\ldots\]

Exceptionals

\[
\begin{array}{cccc}
\ast \\
E_6 \\
(E_{10}) \\
\end{array}
\]

\[
\begin{array}{cccc}
\ast \\
E_7 \\
((A_{16}/2)^t) \\
\end{array}
\]

\[
\begin{array}{cccc}
\ast \\
E_8 \\
(E_{28}) \\
\end{array}
\]

\textbf{Figure 1.} Classification of modules and subgroups of quantum SU(2).
**SU(3)_k**

Orbifold series

\[ A_1 \quad A_2 \quad A_3 \quad A_4 \quad A_5 \quad A_6 \quad \ldots \]

\[ A_1/\beta \quad A_2/\beta \quad A_3/3 \quad A_4/\beta \quad A_5/\beta \quad A_6/3 \quad \ldots \]

Conjugate orbifold series

\[ A_1 \quad A_2 \quad A_3^c \quad A_4^c \quad A_5^c \quad A_6^c \quad \ldots \]

\[ [3A_1^c] [3A_2^c] \quad 3A_3^c \quad 3A_4^c \quad 3A_5^c \quad 3A_6^c \quad \ldots \]

Exceptionals

\[ E_5 \quad E_5/3 = (E_5)^c \quad E_9 \quad E_9/3 = (E_9)^c \quad (A\psi 3)^I \quad (A\psi 3)^{Ic} \quad E_{21} \]

**Figure 2.** Classification of modules and subgroups of quantum SU(3).
\( \text{SU}(4)_k \)

Orbifold series  
\[
\begin{align*}
A_1 & \quad A_2 & \quad A_3 & \quad A_4 & \quad \ldots \\
A_{1/2} & \quad A_{2/2} & \quad A_{3/2} & \quad A_{4/2} & \quad \ldots
\end{align*}
\]

Conjugate orbifold series  
\[
\begin{align*}
A_1/2 & \quad A_2^c & \quad A_3^c & \quad A_4^c & \quad \ldots \\
[2A_1^c] & \quad [2A_2^c] & \quad [2A_3^c] & \quad [2A_4^c] & \quad \ldots
\end{align*}
\]

Figure 3
Figure 4. Classification of modules and subgroups of quantum $SU(4)$. 

\[ \text{Exceptionals} \]

\[ A_2/4 \quad A_6/4 \quad A_8/4 \quad \ldots \quad \begin{array}{c}
A_2 \\
[2(A_2^c/2)] \\
2(A_6^c/2) \\
2(A_8^c/2) \\
\ldots
\end{array} \]

\[ E_4 \quad E_6 \quad E_6/5 = (E_6)^c \quad E_8 \quad E_8/2 = (E_8)^c \text{ (A}_8/4)_I \text{ (A}_8/4)^{Ic} \]
Figure 2. Modules of exceptionals.

\[ (8, 8) = \mathbb{E}_8 \]
\[ (6, 9) = \mathbb{E}_9 \]
\[ (5, 8) = \mathbb{E}_5 \]

Modules of exceptionals

\[ \text{SU}(4) \]
\[ \text{SU}(3) \]
Exceptionally twisted modules of orbifolds.
Figure 7. Modular ladder for the $D_5$ graph.
Figure 8. Modular ladder for the $E_6$ graph.
Figure 9. Modular ladder for the $E_7^c$ graph.
Figure 10
Figure 11. Modular ladder for the $E_8$ graph.